Book 2

Year 10

Mathematics

Mathematics

Year 10 Book Two

GOVERNMENT OF SĀMOA
MINISTRY OF EDUCATION, SPORTS AND CULTURE

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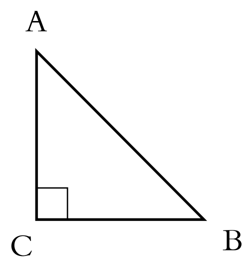
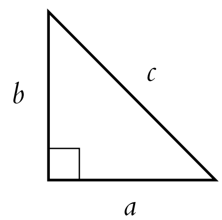
Section 6.1 Using Pythagoras' Theorem With Right-Angled Triangles

What is Pythagoras' Theorem?

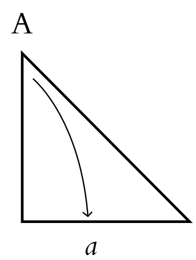
Pythagoras' Theorem gives the relationship between the length of the hypotenuse and the lengths of the other two sides of a right-angled triangle. It can be used to find one side length of a right-angled triangle when the other two side lengths are known.

A right-angled triangle can be labelled in two ways:





1. Label the vertices (corners) (use upper case letters – or capital letters)
2. Label the sides (use lower case letters – or small letters)

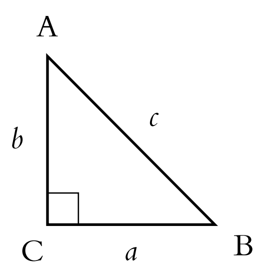


Notice that the lower case letter is opposite the upper case letter.

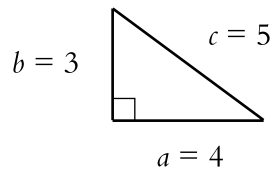
Pythagoras' Theorem

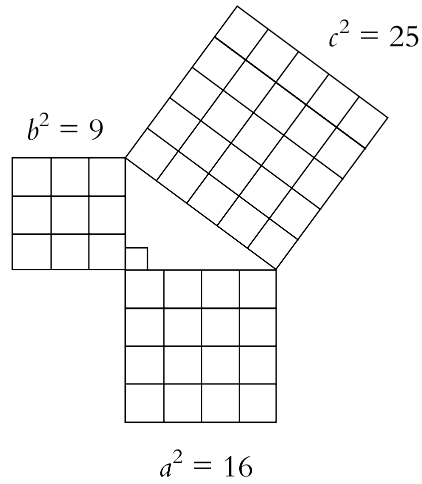
In a right-angled triangle

$c^2 = a^2 + b^2$



Pythagoras' Theorem can be shown as:



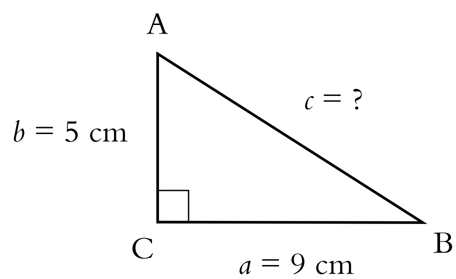


$$\begin{aligned} c^2 &= a^2 + b^2 \\ 5^2 &= 4^2 + 3^2 \\ 25 &= 16 + 19 \end{aligned}$$

Calculating the Length of the Hypotenuse

Example

Calculate the length of the side AB of this triangle.



Solution

In this triangle

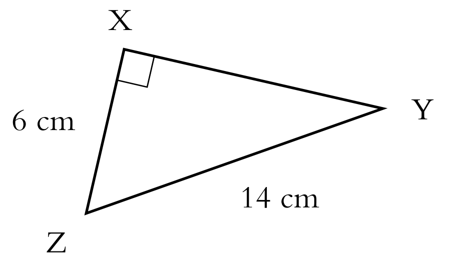
$$\begin{aligned} AB^2 &= BC^2 + AC^2 \text{ or } c^2 = a^2 + b^2 \\ &= 9^2 + 5^2 \\ &= 81 + 25 \\ &= 106 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{106} = 10.29563014 \text{ cm} \\ &= 10.3 \text{ cm (to 1 decimal place)} \end{aligned}$$

Calculating the Length of the Other Sides

Example

Calculate the length of the side XY of this triangle.



Solution

In this triangle,

$YZ^2 = XY^2 + XZ^2$

$14^2 = XY^2 + 6^2$

$196 = XY^2 + 36$

$XY^2 = 160$

$XY = \sqrt{160}$

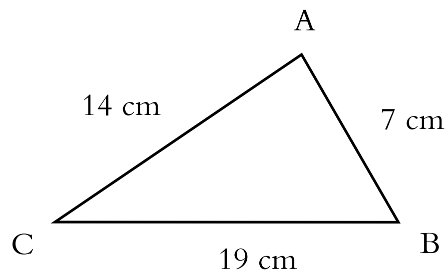
$= 12.64911064$ cm

$= 12.6$ cm (to 1 decimal place)

Solving Problems using Pythagoras' Theorem

Example

Determine whether or not this triangle contains a right angle.



Solution

If the triangle does contain a right angle, then the longest side, BC, would be the hypotenuse. So, the triangle will be right-angled if $AB^2 + AC^2 = BC^2$.

The two sides

$AB^2 + AC^2 = 7^2 + 14^2$

$= 49 + 196$

$= 245$

The hypotenuse

$BC^2 = 19^2$

$= 361$

In this triangle

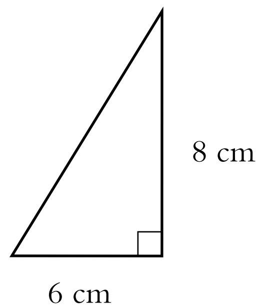
$AB^2 + AC^2 \neq BC^2$

so it does not contain a right angle.

Skill Exercises: Using Pythagoras' Theorem.

1. Calculate the length of the hypotenuse of each of the triangles shown. Where necessary, give your answers correct to 2 decimal places.

(a)



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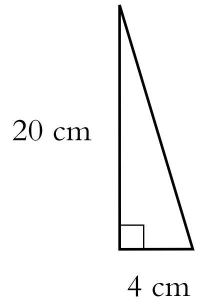
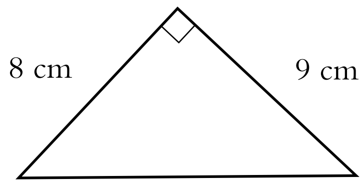
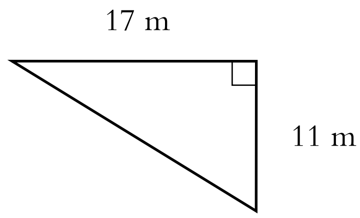


(c)

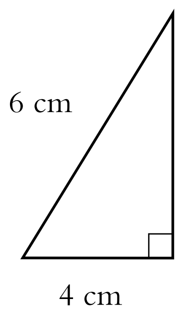


(d)

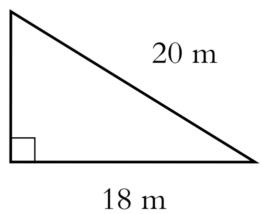


2. Calculate the length of the unmarked side of each of the triangles shown. In each case, give your answer correct to 2 decimal places.

(a)



(b)



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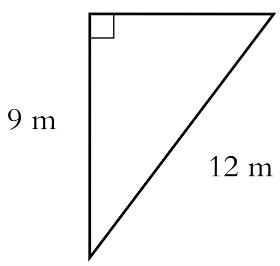

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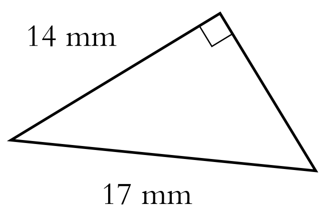
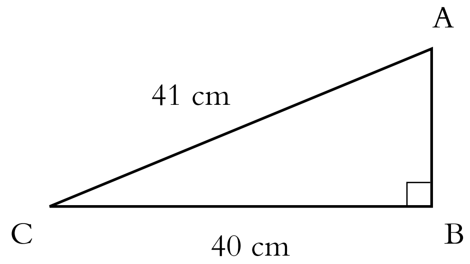
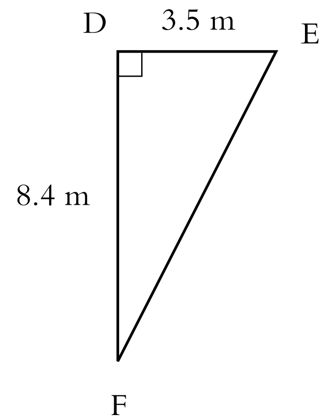


3. Calculate:

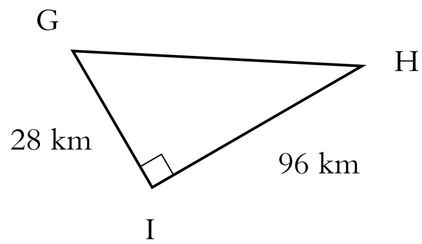
(a) Calculate AB



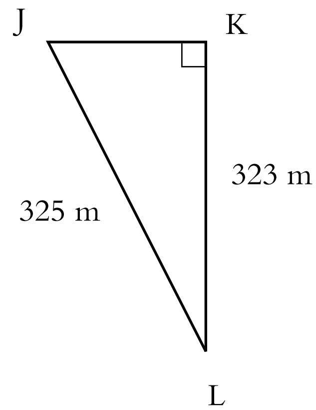
(b) Calculate EF



(c) Calculate GH

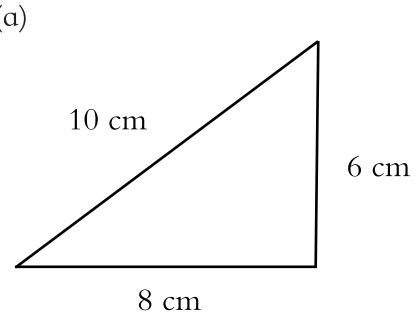


(d) Calculate JK



4. Which of the triangles below contain right angles?

(a)



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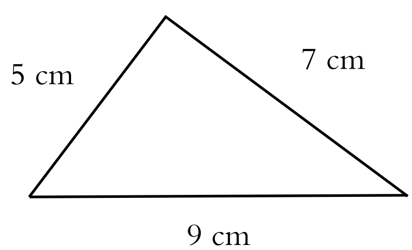

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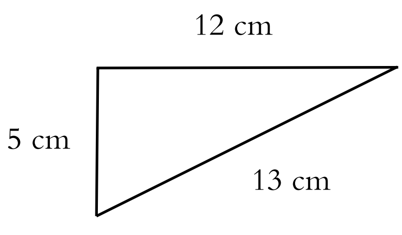

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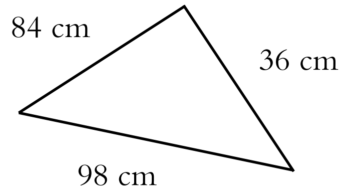
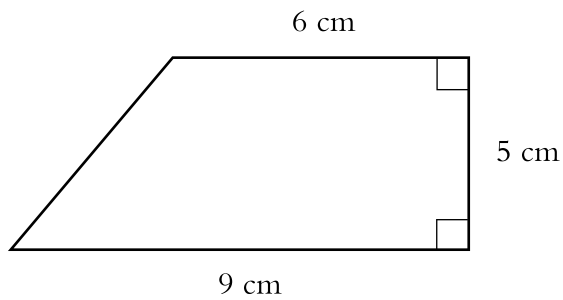


5. Sinapi walks 100 m north and then 100 m east. How far is she from her starting position? Give your answer to a sensible level of accuracy.

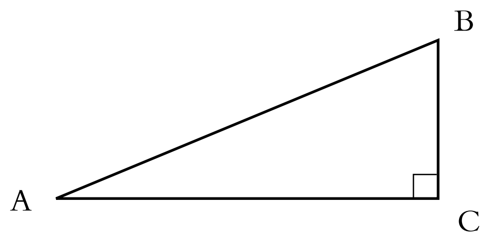
6. Calculate the perimeter of the trapezium shown. Give you answer to the nearest centimetre.



7. The diagram shows a plan for a wheelchair ramp.

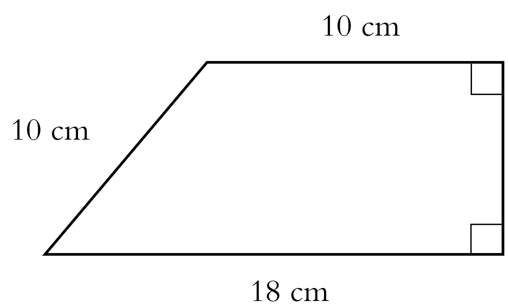
   The distance AC is 2 m.

   Giving your answer in metres, correct to the nearest cm, calculate the distance AB if:



   (a) BC = 20 cm (b) BC = 30 cm

8. Calculate the perimeter and area of this trapezium.



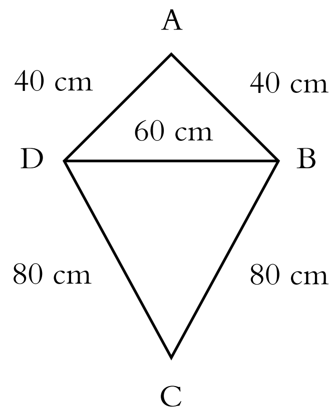
9. A rope is 10 m long. One end is tied to the top of a flagpole. The height of the flagpole is 5 m. The rope is pulled tight with the other end on the ground.

   How far is the end of the rope from the base of the flagpole? Give your answer to a sensible level of accuracy.

10. A ladder leans against a vertical wall. The length of the ladder is 5 m. The foot of the ladder is 2 m from the base of the wall.

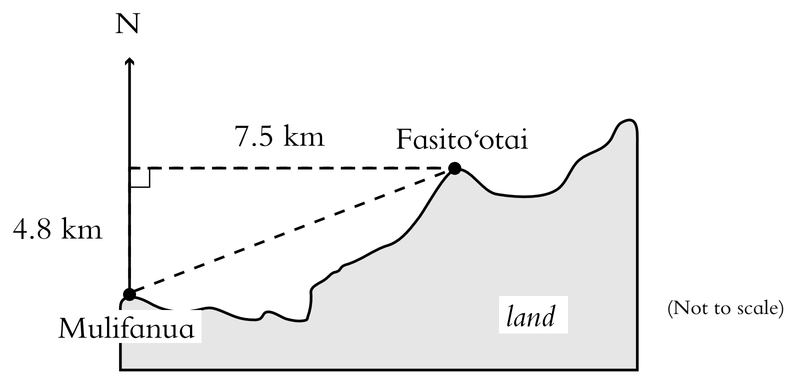
    How high is the top of the ladder above the ground? Give your answer to a sensible level of accuracy.

11. Sala makes a kite from two isosceles triangles, as shown in the diagram.

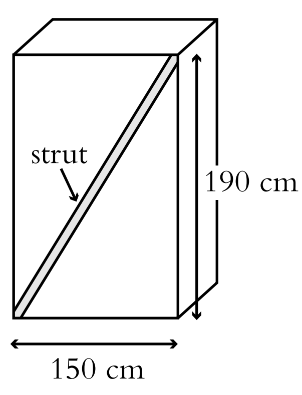


Calculate the height, AC, of the kite, giving your answer to the nearest centimetre.

12. Fasito‘otai is 7.5 km east and 4.8 km north of Mulifanua. Calculate the direct distance from Mulifanua to Fasito‘otai. Show your working.

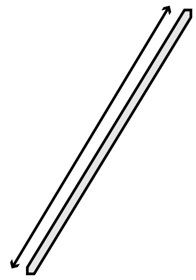


13. A cupboard needs to be strengthened by putting a strut on the back of it like this.

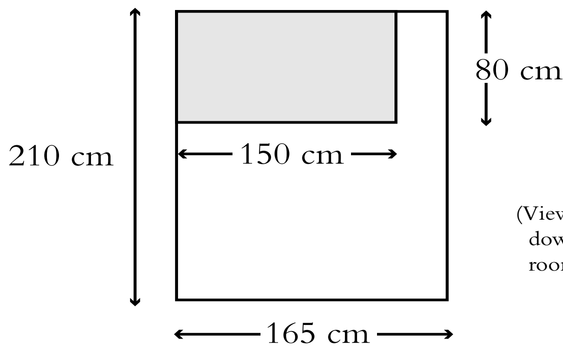


(a) Calculate the length of the diagonal strut.

Show your working.

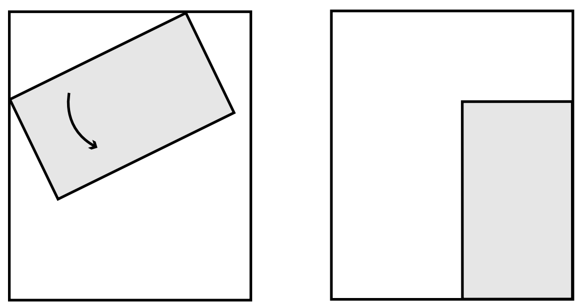


(b) In a small room the cabinet is in this position:



(View looking down on the room)

Calculate if the room is wide enough to turn the cabinet like this and put it in its new position. Show your working.



(New position)

Section 6.2 Using Trigonometric Ratios With Right-Angled Triangles

Defining Trigonometric Ratios

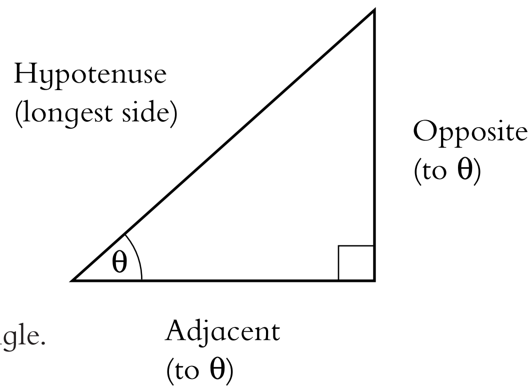
A Trigonometric Ratio describes the relationship between the size of one angle and the lengths of two sides in a right angle triangle.

In this section, three trigonometric ratios are used.

They are:	sine	(shortened to sin)
	cosine	(shortened to cos)
	tangent	(shortened to tan)

The three sides of a right-angled triangle have special names.

1. The adjacent side is the side that joins both the angle and the right angle.
2. The opposite side is opposite the angle.
3. The hypotenuse is the side opposite the right angle and is the longest side in the triangle.



Using these definitions, we can write down the trigonometric functions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{O}{H} \quad \text{(SOH)}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H} \quad \text{(CAH)}$$

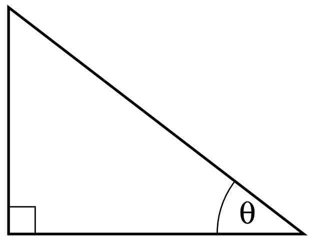
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A} \quad \text{(TOA)}$$

The three ratios can be remembered by using: SOHCAHTOA

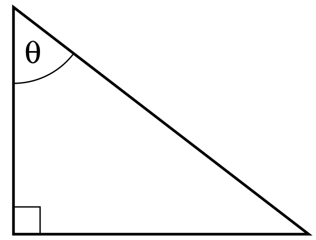
Example 1

Label the sides of these right-angled triangles:

(a)



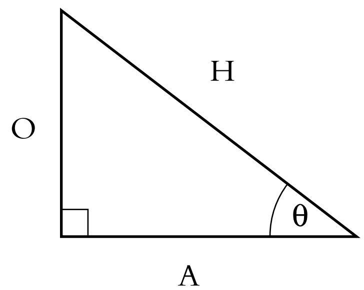
(b)



Solution

Label the hypotenuse (longest) side first, then label the other two sides.

(a)



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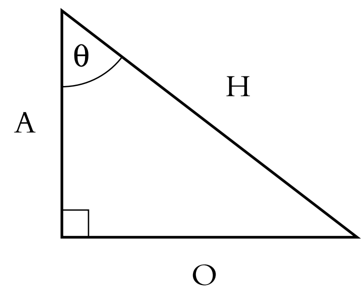
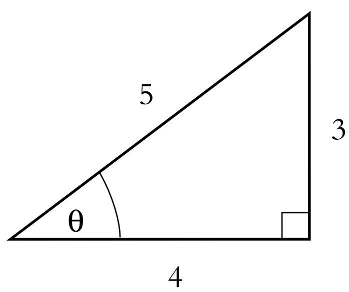


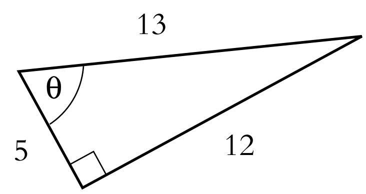
Example 2

Write the three trigonometric ratios for these right-angled triangles.

(a)



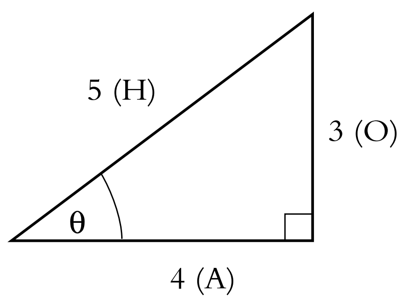
(b)



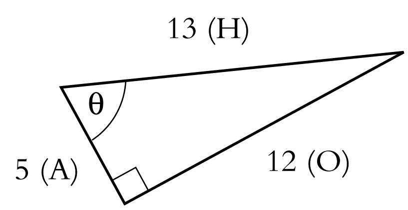
Solution

1. Label the sides.

(a)



(b)



2. Use SOHCAHTOA to write the three ratios.

(a)

$$S = \frac{O}{H}$$

$$\sin\theta = \frac{3}{5}$$

$$C = \frac{A}{H}$$

$$\cos\theta = \frac{4}{5}$$

$$T = \frac{O}{A}$$

$$\tan\theta = \frac{3}{4}$$

(b)

$$S = \frac{O}{H}$$

$$\sin\theta = \frac{12}{13}$$

$$C = \frac{A}{H}$$

$$\cos\theta = \frac{5}{13}$$

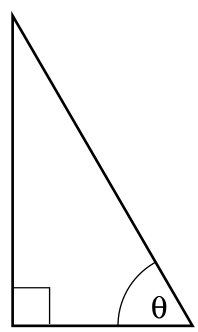
$$T = \frac{O}{A}$$

$$\tan\theta = \frac{12}{5}$$

Skill Exercises: Defining Trigonometric Ratios

1. Label the sides of these right-angled triangles:

(a)



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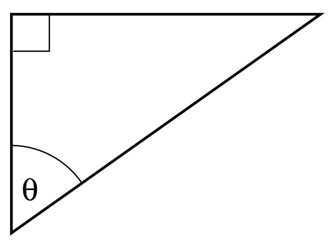

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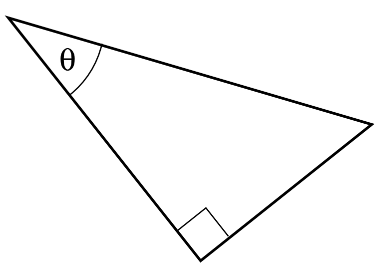

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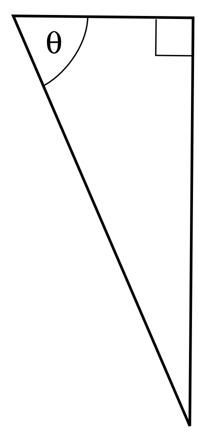
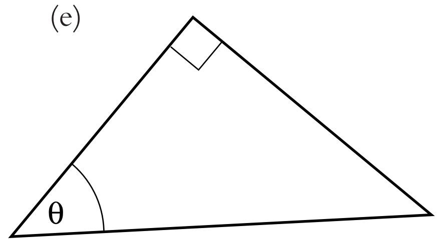
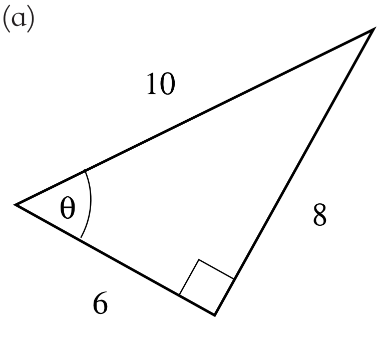


(e)

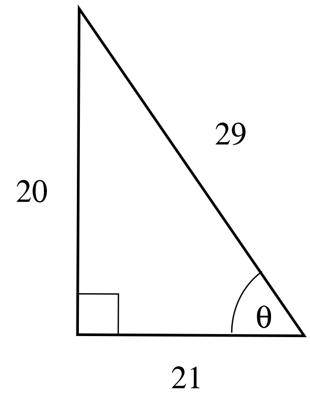


2. Write the three trigonometric ratios for these triangles:

(a)



(b)



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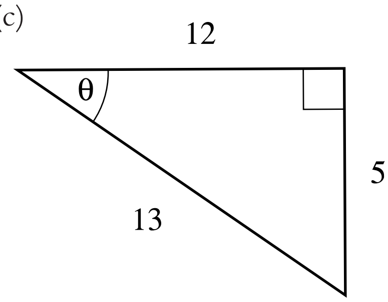
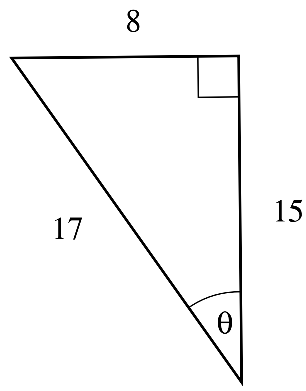
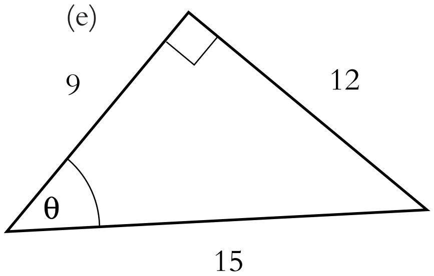


(d)



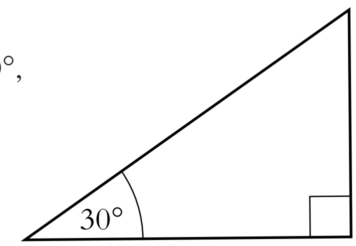
(e)



Estimating Trigonometric Ratios

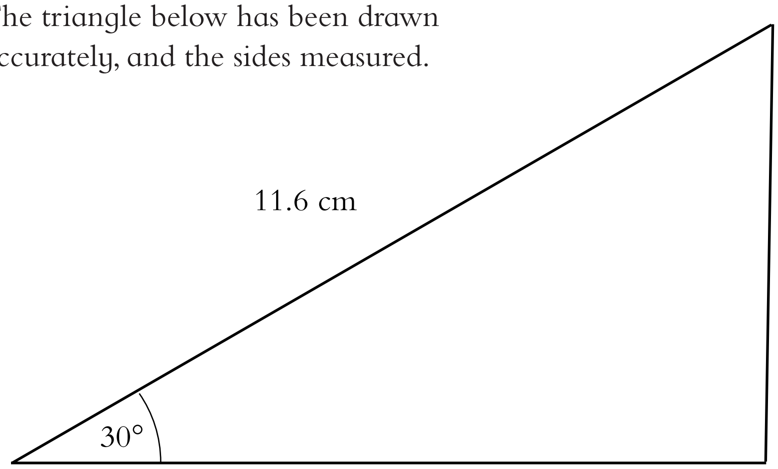
Example 1

Estimate the sin, cos and tan of 30°, using an accurate drawing of the triangle shown.



Solution

The triangle below has been drawn accurately, and the sides measured.



Here, hypotenuse = 11.6 cm, adjacent = 10 cm and opposite = 5.8 cm, so:

$$\sin 30° = \frac{O}{H} = \frac{5.8}{11.6} = 0.5$$

$$\cos 30° = \frac{A}{H} = \frac{10}{11.6} = 0.86 \text{ (to 2 decimal places)}$$

$$\tan 30° = \frac{O}{A} = \frac{5.8}{10} = 0.58$$

You can obtain more accurate values of sin 30°, cos 30° and tan 30° by using a scientific calculator. The $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ calculator keys convert the trigonometric ratio back into the angle:

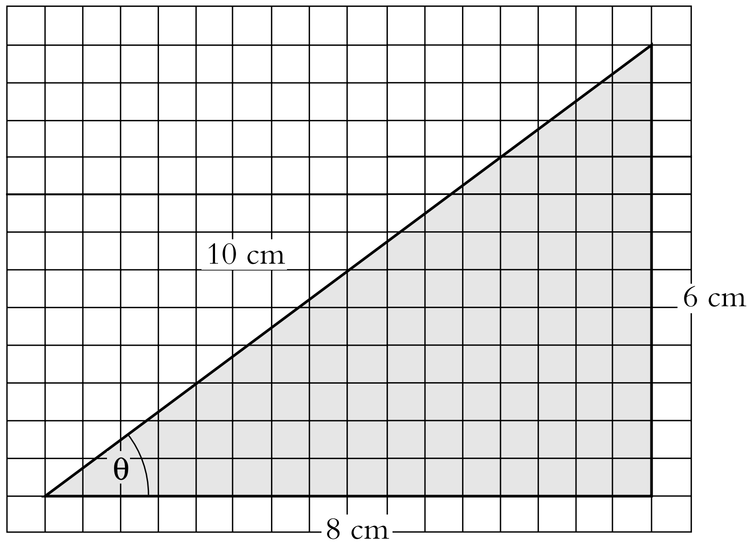
$$\sin 30° = 0.5$$

$$\sin^{-1} 0.5 = 30$$

Warning: When you use a scientific calculator, always check that it is dealing with angles in degree mode.

Example 2

(a) Measure the angle marked in the following triangle:



(b) Calculate the sine, cosine and tangent of this angle.

Solution

(a) In this case the angle can be measured with a protractor as 37°.

(b) Here we have

opposite = 6 cm

adjacent = 8 cm

hypotenuse = 10 cm

$$\sin\theta = \frac{O}{H} = \frac{6}{10} = 0.6 \qquad \cos\theta = \frac{A}{H} = \frac{8}{10} = 0.8 \qquad \tan\theta = \frac{O}{A} = \frac{6}{8} = 0.75$$

Skill Exercises: Estimating Trigonometric Ratios

1. (a) Draw three different right-angled triangles that each contain a 60° angle.

   (b) Use each triangle to estimate sin 60°, and check that you get approximately the same value in each case.

   (c) Estimate a value for cos 60°.

   (d) Estimate a value for tan 60°.

2. (a) Draw a right-angled triangle that contains an angle of $50°$.
   (b) Use this triangle to estimate:
   (i) $\cos 50°$ (ii) $\sin 50°$ (iii) $\tan 50°$

3. (a) Draw a right-angled triangle which contains a $45°$ angle.
   (b) Explain why $\sin 45° = \cos 45°$ and state the value of $\tan 45°$.

4. (a) Copy and complete the following table, giving you values correct to 2 significant figures.

   Draw appropriate right-angled triangles to be able to estimate the values.

Angle	sine	cosine	tangent
10°			
20°			
30°			
40°			
50°			
60°			
70°			
80°			

   (b) Use the sin, cos and tan keys on your calculator to check your values.

5. A student says that the sine of an angle is 0.5. What is the angle?

6. If the cosine of an angle is 0.17, what is the angle? Give the most accurate answer you can obtain from your calculator and then round it to the nearest degree.

7. What are the values of:
   (a) $\cos 0°$ (b) $\sin 0°$ (c) $\sin 90°$
   (d) $\cos 90°$ (e) $\tan 0°$ (f) $\tan 90°$

8. Use your calculator to obtain the following, correct to 3 significant figures:
   (a) $\sin 82°$ (b) $\cos 11°$ (c) $\sin 42°$
   (d) $\tan 80°$ (e) $\tan 52°$ (f) $\tan 38°$

9. Use your calculator to obtain the angle θ, correct to 1 decimal place, if:
   (a) $\cos \theta = 0.3$ (b) $\sin \theta = 0.77$ (c) $\tan \theta = 1.62$
   (d) $\sin \theta = 0.31$ (e) $\cos \theta = 0.89$ (f) $\tan \theta = 11.4$

10. A student calculates that $\cos\theta = 0.8$.

    (a) By considering the sides of a suitable right-angled triangle, work out the values of $\sin\theta$ and $\tan\theta$.

    (b) Use a calculator to find the angle θ.

    (c) Use the angle you found in part (b) to verify your answers to part (a).

Calculating the Length of a Side

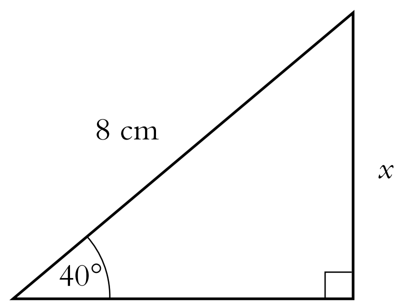
In this section we use the trigonometric ratios to calculate the lengths of sides in a right-angled triangle.

Trigonometric Ratios

$$\sin\theta = \frac{O}{H} \qquad \cos\theta = \frac{A}{H} \qquad \tan\theta = \frac{O}{A}$$

Example 1

Calculate the length of the side marked x in this triangle:



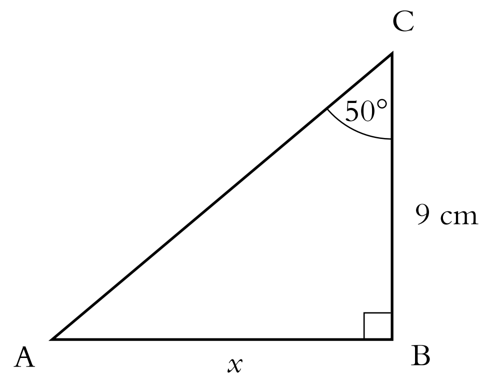
Solution

This question uses the opposite side and the hypotenuse. These two sides appear in the ratio for $\sin\theta$:

$$\begin{aligned}
\sin\theta &= \frac{O}{H} \\
\sin 40^\circ &= \frac{x}{8} \\
x &= 8 \times \sin 40^\circ \\
&= 5.142300877 \text{ cm} \\
&= 5.1 \text{ cm to 1 decimal place}
\end{aligned}$$

Example 2

Calculate the length of the side AB of this triangle.



Solution

Side AB is the opposite side and side BC is the adjacent side, ∴ use the formula:

$$\tan\theta = \frac{O}{A}$$

$$\tan 50° = \frac{x}{9}$$

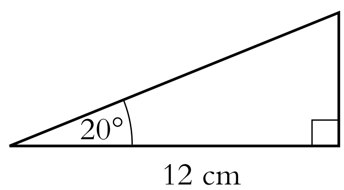
$$x = 9 \times \tan 50°$$

$$= 10.72578233 \text{ cm}$$

$$= 10.7 \text{ cm to 1 decimal place}$$

Example 3

Calculate the length of the hypotenuse of this triangle:



Solution

The trigonometric ratio that links the adjacent side and the hypotenuse is the cosine ratio.

$$\cos\theta = \frac{A}{H}$$

$$\cos 20° = \frac{12}{H}$$

$$H \times \cos 20° = 12$$

$$H = \frac{12}{\cos 20°}$$

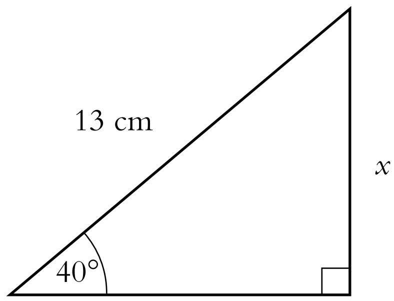
$$= 12.77013327 \text{ cm}$$

Therefore the hypotenuse length is 12.8 cm to 1 decimal place.

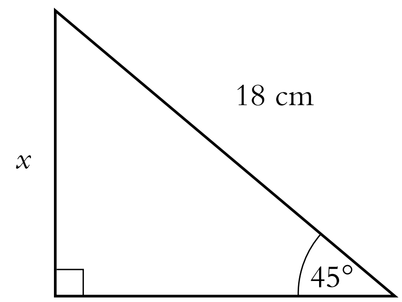
Skill Exercises: Calculating the Length of a Side

1. Use the ratio for sine to work out the length of the side marked x in each of the following triangles. In each case, give your answer correct to 1 decimal place.

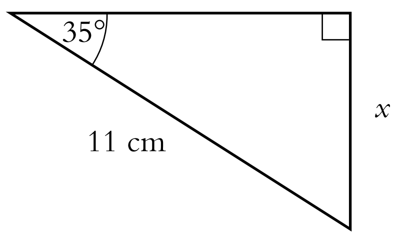
(a)



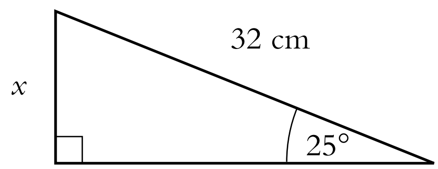
(b)



(c)

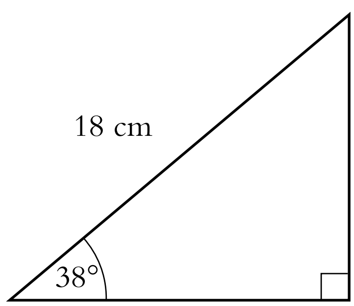


(d)

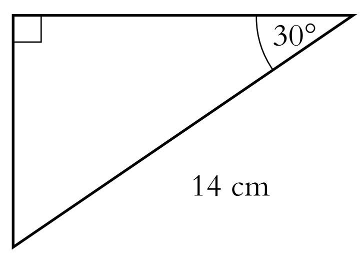


2. Use the ratio for cosine to find the length of the adjacent side in each of the following triangles. Give your answers correct to 1 decimal place.

(a)



(b)



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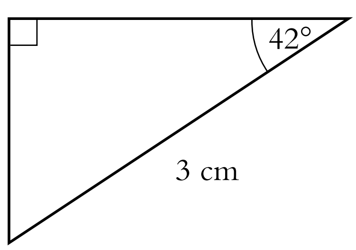
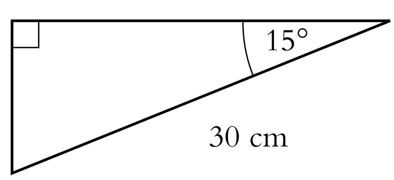
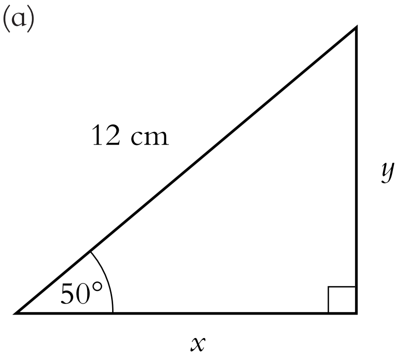


(d)

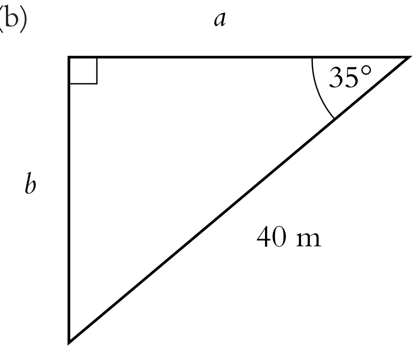


3. Calculate the length of sides indicated by letters in each of the following triangles. Give each of your answers correct to 3 significant figures.

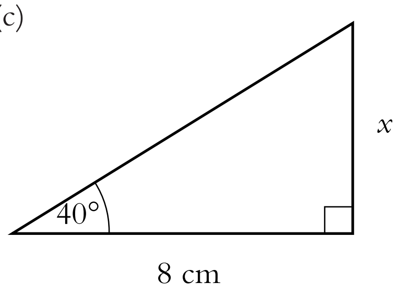
(a)



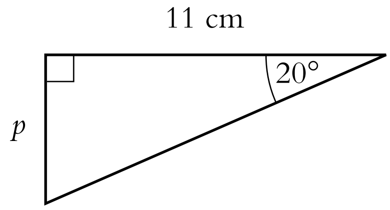
(b)



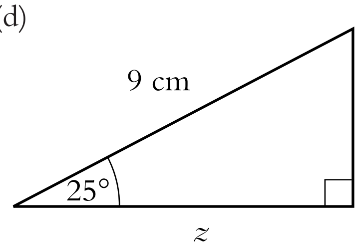
(c)



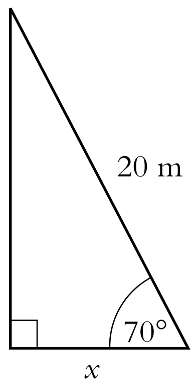
(d)



(d)

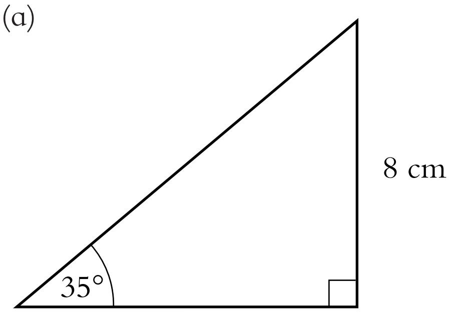


(f)

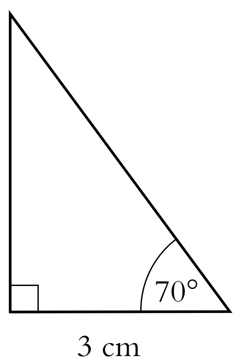


4. Calculate the length of the hypotenuse of each of the following triangles. Give each of your answers correct to 3 significant figures.

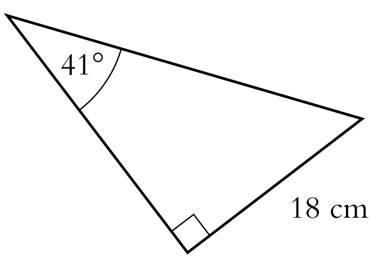
(a)



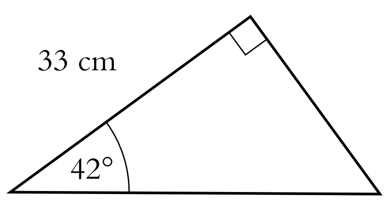
(b)



(c)

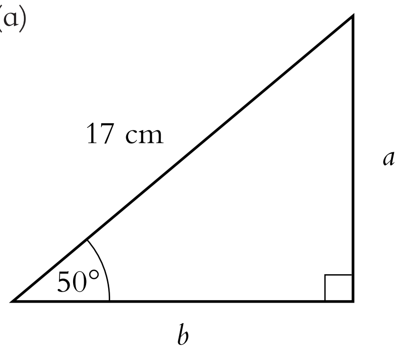


(d)

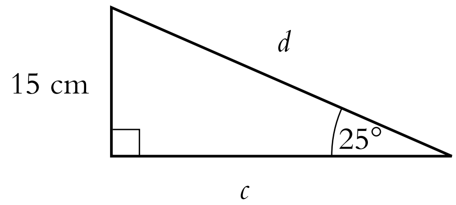


5. Calculate all the lengths marked with letters in the following triangles. Give each of your answers correct to 2 decimal places.

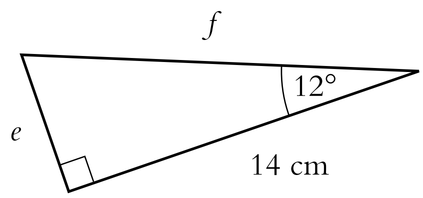
(a)



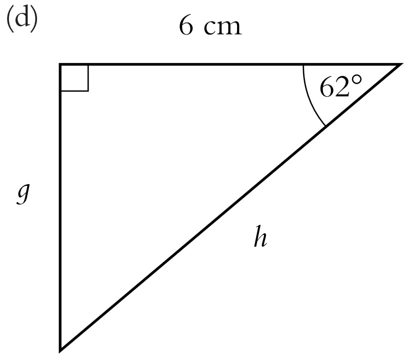
(b)



(c)



(d)



6. A ladder, which has length 6 m, leans against a vertical wall. The angle between the ladder and the horizontal ground is 65°.

   (a) How far is the foot of the ladder from the wall?

   (b) What is the height of the top of the ladder above the ground?

   In each case, give your answer to the nearest centimetre.

7. A boat sails 50 km on a bearing of 070°.

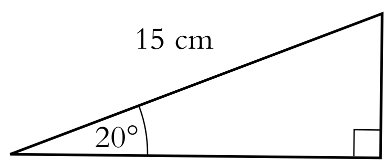
   (a) How far east does the boat travel?

   (b) How far north does the boat travel?

   In each case, give your answer to a sensible level of accuracy.

8. Calculate the perimeter and area of this triangle:

   Give your answers correct to 2 decimal places.



9. A ramp has length 6 m and is at an angle of 50° above the horizontal. How high is the top of the ramp? Give your answer to a sensible level of accuracy.

10. A rope is stretched from a window on a building to a point on the ground, 6 m from the base of the building. The angle between the rope and the side of the building is 19°.

    (a) How long is the rope?

    (b) How high is the window?

    In each case, give your answer correct to the nearest centimetre.

Calculating the Size of an Angle

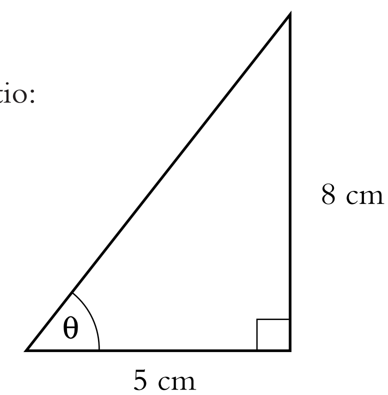
In this section we use trigonometry to determine the sizes of angles in right-angled triangles. On your scientific calculator are buttons labelled $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Use these to calculate the angles in the problems which follow:

Trigonometric Ratios

$$\sin\theta = \frac{O}{H} \qquad \cos\theta = \frac{A}{H} \qquad \tan\theta = \frac{O}{A}$$

Example 1

Calculate the angle θ in this triangle:



Solution

The given lengths are the adjacent and opposite sides, therefore use the tangent ratio:

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{8}{5}$$

$$= 1.6$$

Use the $\tan^{-1}$ key on a calculator to obtain

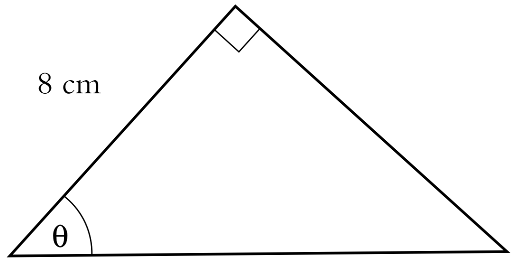
$$\theta = \tan^{-1}(1.6)$$

$$= 57.99461678°$$

$$= 58.0° \text{ (to 1 decimal place)}$$

Example 2

Calculate the angle marked θ in this triangle:



Solution

The lengths given are the adjacent side and the hypotenuse. Therefore use the cosine ratio:

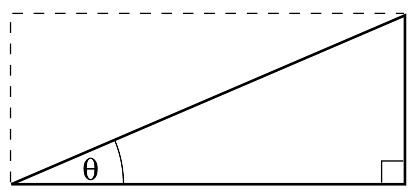
$$\begin{aligned}\cos\theta &= \frac{A}{H}\\ &= \frac{8}{17}\\ &= 0.470588235\\ \theta &= \cos^{-1}(0.470588235)\\ &= 61.92751306^\circ\\ &= 61.9^\circ \text{ (to 1 decimal place)}\end{aligned}$$

Example 3

A rectangle has sides of lengths 5 m and 10 m. Determine the angle between the long side of the rectangle and a diagonal.

Solution

The solution is illustrated in this diagram:



Using the tangent ratio gives:

$$\begin{aligned}\tan\theta &= \frac{5}{10}\\ &= 0.5\\ \theta &= \tan^{-1}(0.5)\\ &= 26.56505118^\circ\\ &= 26.6^\circ \text{ (to 1 decimal place).}\end{aligned}$$

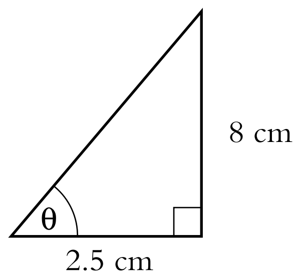
Skill Exercises: Calculating the Size of an Angle

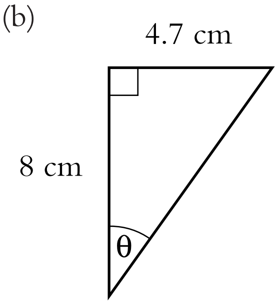
1. Giving your answers, where necessary, correct to 1 decimal place, use your calculator to obtain θ if:

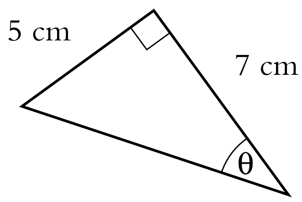
   (a) $\sin\theta = 0.8$ (b) $\cos\theta = 0.5$ (c) $\tan\theta = 1$

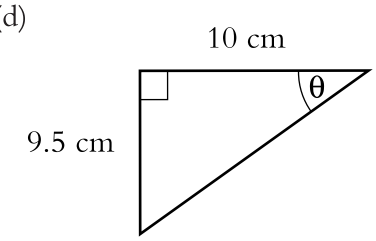
   (d) $\sin\theta = 0.3$ (e) $\cos\theta = 0$ (f) $\tan\theta = 14$

2. Use the tangent function to calculate the angle θ in each of the following diagrams. In each case, give your answer correct to 1 decimal place.

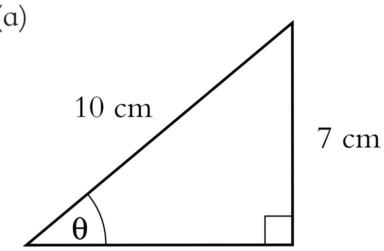
   (a)


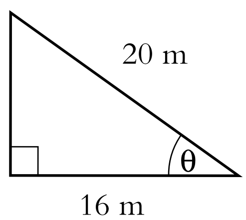
   (b)


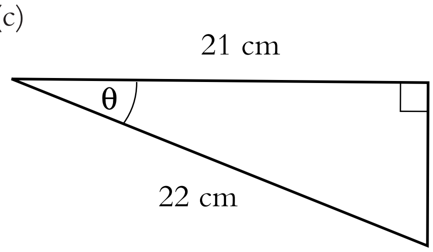
   (b)


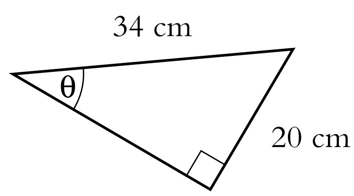
   (d)


3. Use sine or cosine to calculate the angle θ in each of the following triangles. In each case, give your answer correct to 1 decimal place.

   (a)


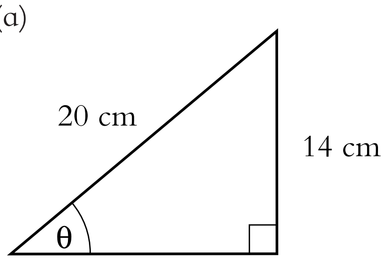
   (b)


   (c)


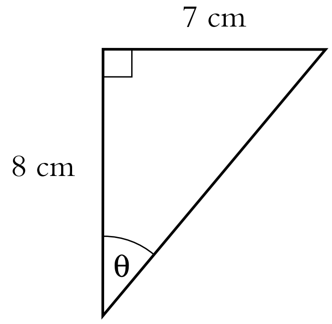
   (d)


4. Calculate the angle θ in each of the following triangles. In each case, give your answer correct to 1 decimal place.

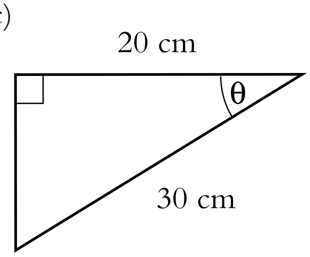
(a)



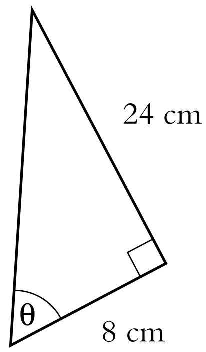
(b)



(c)



(d)

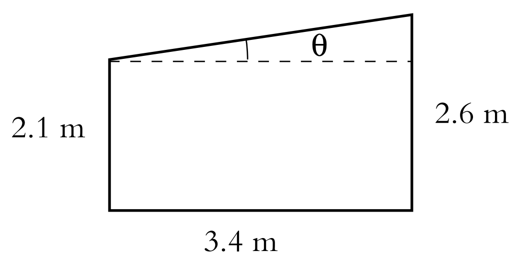


5. A right-angled triangle has sides of length 3 cm, 4 cm and 5 cm.

Find the sizes of all the angles in the triangle, giving your answers to the nearest degree.

6. The diagram shows the cross-section of a shed.

Calculate the angle θ between the roof and the horizontal.
Give your answer to the nearest degree.



7. A ladder of length 6 m leans against a wall. The foot of the ladder is at a distance of 3 m from the base of the wall.

Calculate the angle between the ladder and the ground.

8. A rectangle has sides of length 12 cm and 18 cm.

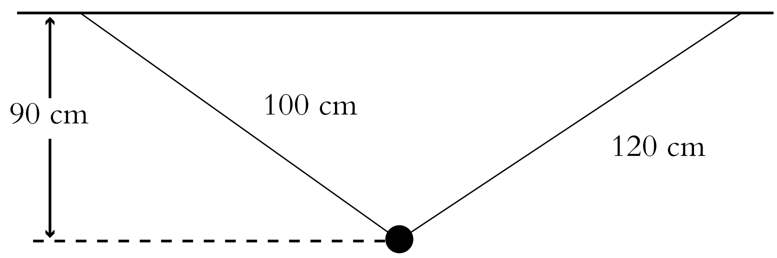
(a) Calculate the length of the diagonal of the rectangle, giving your answer to the nearest millimetre.

(b) Calculate the angle between the diagonal and the shorter side of the rectangle, giving your answer to the nearest degree.

9. As an aeroplane travels 3000 m along a straight flight path, it rises 500 m.

   Calculate the angle between the flight path of the aeroplane and the horizontal. Give your answer to a sensible level of accuracy.

10. A weight hangs from two strings as shown in the diagram.



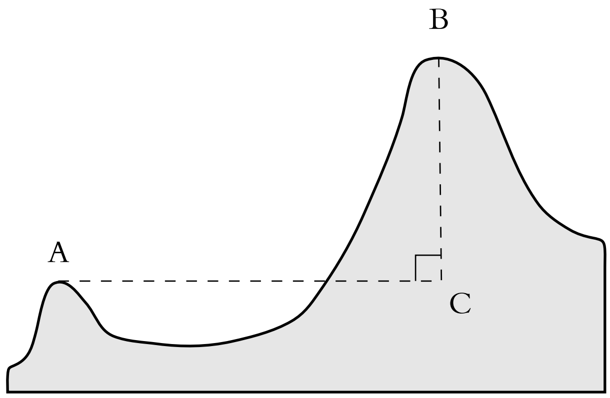
   Calculate the angle between the two strings, giving your answer to the nearest degree.

Solving Problems using Trigonometric Ratios

Trigonometry is a branch of mathematics that is used in engineering, architecture, surveying, navigation and astronomy.

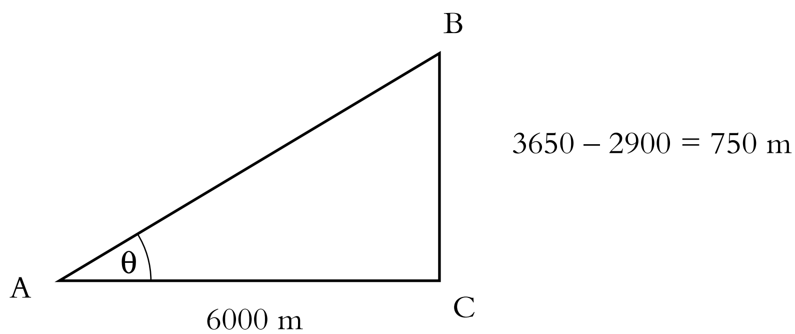
Example 1

A and B are points on two hilltops. The distance from A to C is 6 km.



The height of A is 2900 m and the height of B is 3650 m. Find the angle of elevation from A to B.

Solution



The known side lengths are the opposite and adjacent sides. Therefore use the tangent ratio:

$$T = \frac{O}{A}$$

$$\tan\theta = \frac{750 \text{ m}}{6000 \text{ m}}$$

$$\tan\theta = 0.125$$

$$\theta = \tan^{-1}(0.125)$$

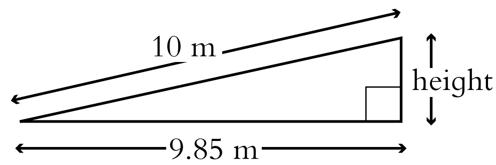
$$= 7.125016349$$

$$= 7° \text{ (to 1 significant figure)}$$

Skill Exercises: Solving Problems using Trigonometric Ratios

1. Ramps help people going into buildings. A ramp that is 10 m long must not have a height greater than 0.83 m.

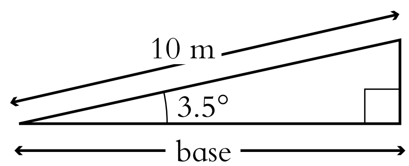
   (a) Here are the plans for a ramp:



   Is this ramp too high?

   You must show calculations to explain your answer.

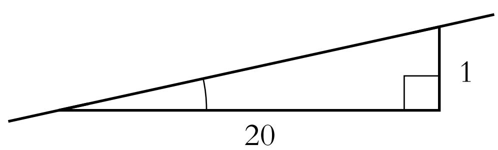
   (b) Here are the plans for a different ramp.



   How long is the base of this ramp?

   You must show your calculations.

   (c) The recommended gradient of a ramp is 1 in 20.

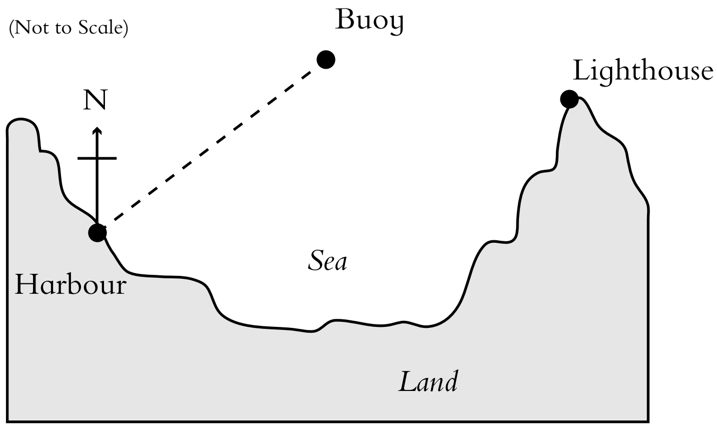


   What angle gives the recommended gradient?

   You must show your calculations.

2. A boat sails from a harbour to a buoy.

   The buoy is 6 km to the east and 4 km to the north of the harbour.



   (a) Calculate the shortest distance between the buoy and the harbour. Give your answer to 1 decimal place.

   Show your working.

   (b) Calculate the bearing of the buoy from the harbour.

   Show your working.

   The buoy is 1.2 km to the north of the lighthouse. The shortest distance from the lighthouse to the buoy is 2.5 km.

   (c) Calculate how far the buoy is to the west of the lighthouse. Give your answer to 1 decimal place.

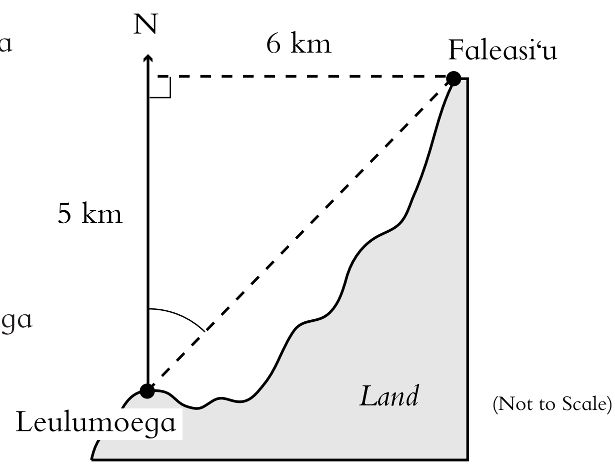
   Show your working.

3. Faleasiʻu is 6 km east and 5 km north of Leulumoega.

   (a) Lauulu wants to sail directly from Leulumoega to Faleasiʻu.

   On what bearing should he sail?

   Show your working.



   (b) Ana sails from Leulumoega on a bearing of 048°. She stops when she is due west of Faleasiʻu.

   How far west of Faleasiʻu is Ana?

   Show your working.

Unit 7: GEOMETRY

In this unit you will be:

7.1 Exploring Transformations using Enlargement

- Drawing an Englargement.
- Finding the Centre of Enlargement.
- Recognising Similar Shapes.
- Working with Line, Area and Volume Ratios.
- Using Enlargement with Maps and Scale Models.

7.2 Exploring Pattern using Line and Rotational Symmetry

- Working with Line Symmetry and Reflection.
- Rotating Shapes and Rotational Symmetry.
- Examining the Symmetry of Polygons.
- Combining Transformations.

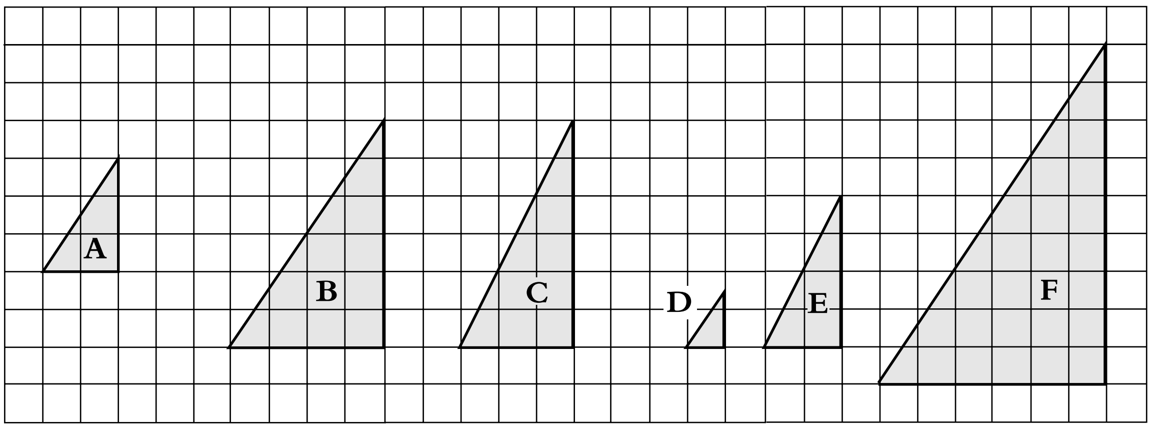
Section 7.1 Exploring Transformations Using Enlargement

Drawing an Enlargement

An enlargement increases or decreases the size of a shape by a multiplier know as the scale factor. The angles in the shape will not be changed by the enlargement.

Example 1

Which of the triangles below are enlargements of the triangle marked A? State the scale factor of each of these enlargements.



Solution

B is an enlargement of A, since all the lengths are doubled.

The scale factor of the enlargement is 2.

C is not an enlargement of A.

D is an enlargement of A, since all the lengths are halved.

The scale factor of the enlargement is $\frac{1}{2}$.

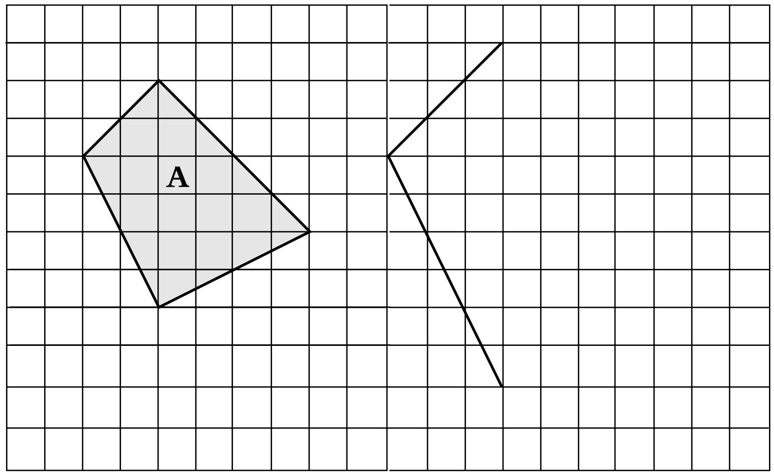
E is not an enlargement of A.

F is an enlargement, since all the lengths are trebled.

The scale factor of the enlargement is 3.

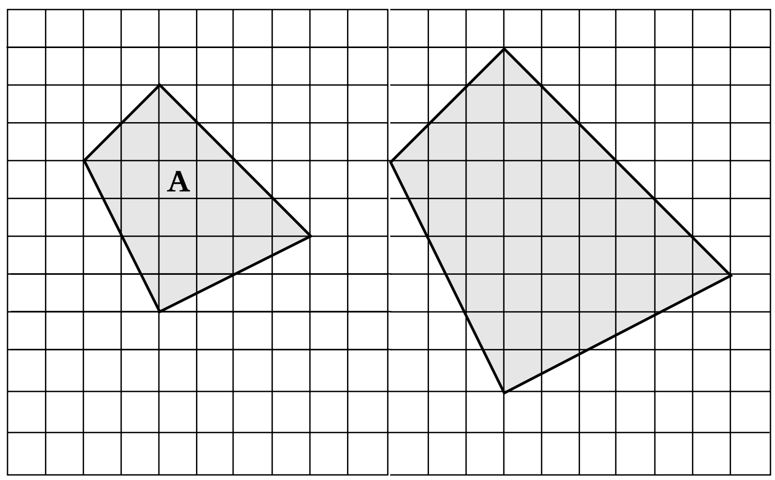
Example 2

Amosa has started to draw an enlargement of the quadrilateral marked A. Copy and complete the enlargement.



Solution

The diagram shows the completed enlargement.

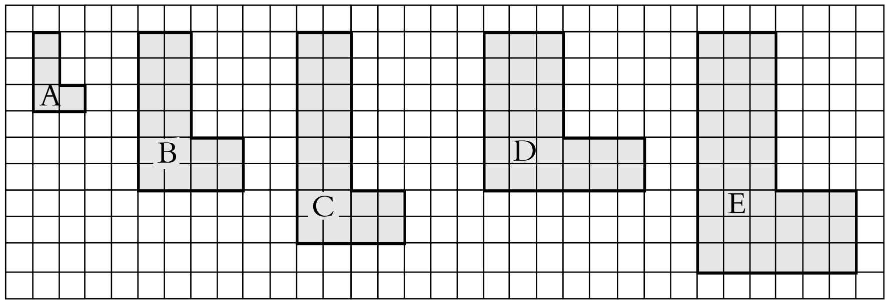


All the lengths have been increased by factor of $1\frac{1}{2}$.

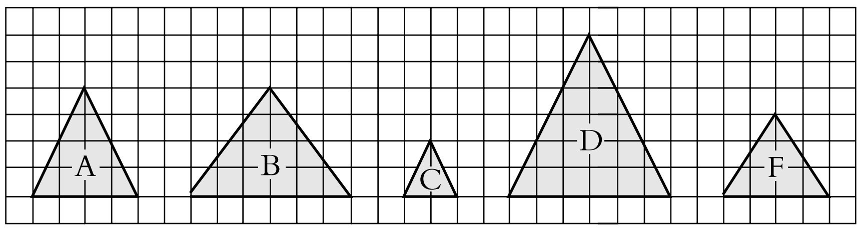
The scale factor of the enlargement is $1\frac{1}{2}$.

Skill Exercises: Drawing an Enlargement

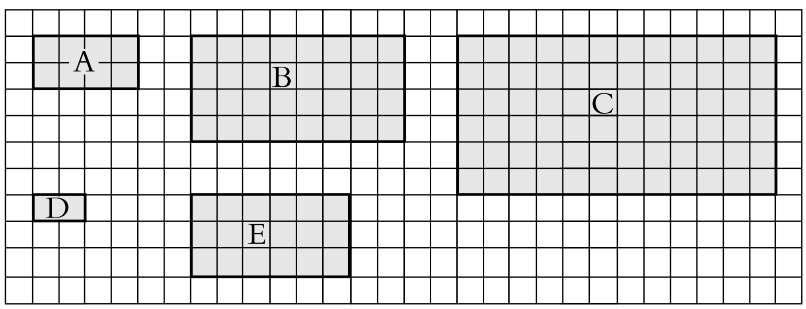
1. Which of the following shapes are enlargements of shape A? State the scale factor of each of these enlargements:



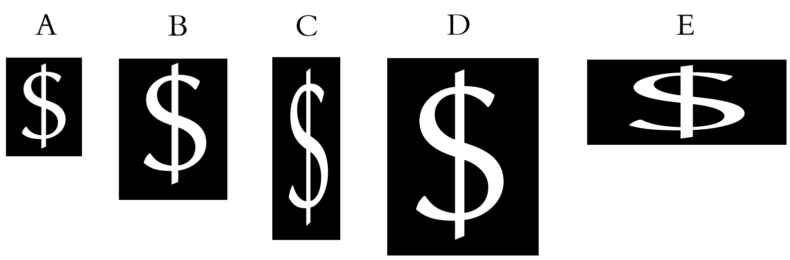
2. Which of the following triangles are **not** enlargements of the triangle marked A?



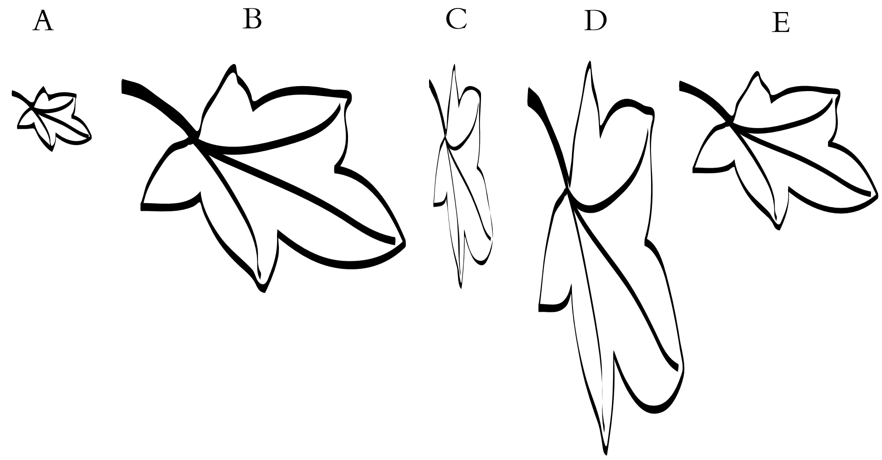
3. The diagram below shows four enlargements of rectangle A. State the scale factor of each enlargement.



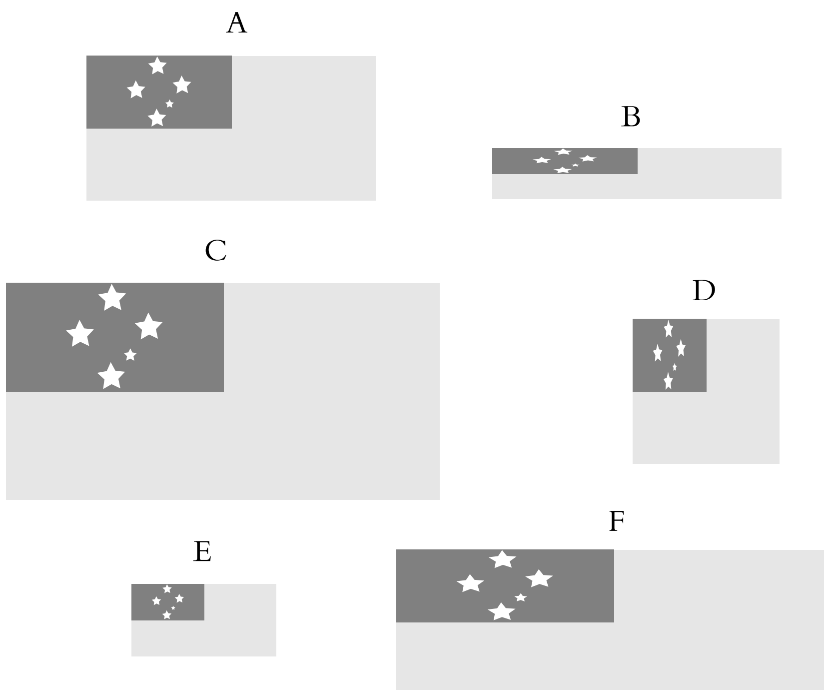
4. Which two signs below are **not** enlargements of sign A?



5. Which two of the leaves shown below are enlargements of leaf A?



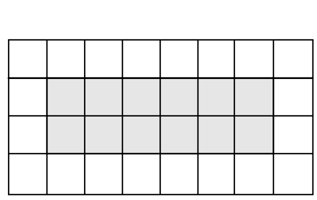
6. Which of the flags below are enlargements of flag A?



7. Draw enlargements of the rectangle shown with scale factors:

   (a) 2 (b) 4

   (c) $\frac{1}{2}$ (d) 3

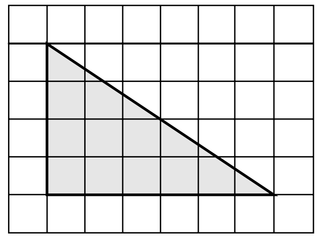


8. Draw enlargements of the triangles shown with scale factors:

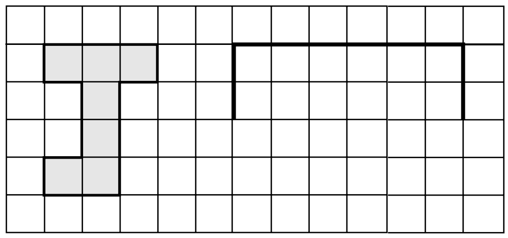
    (a) 2

    (b) 3

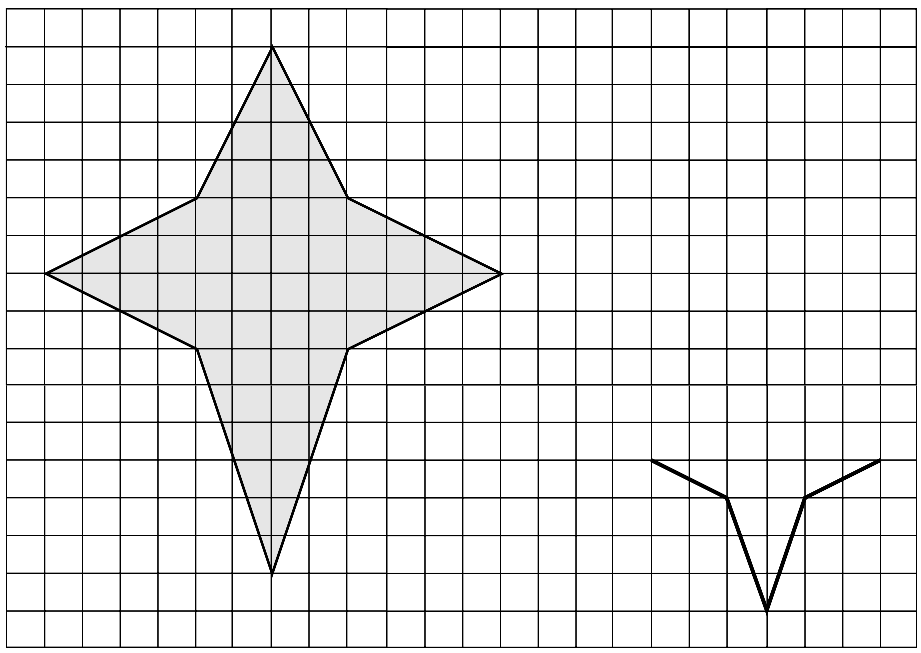
    (c) $1\frac{1}{2}$



9. Perelini has started to draw an enlargement of the shape below. Copy and complete her enlargement.

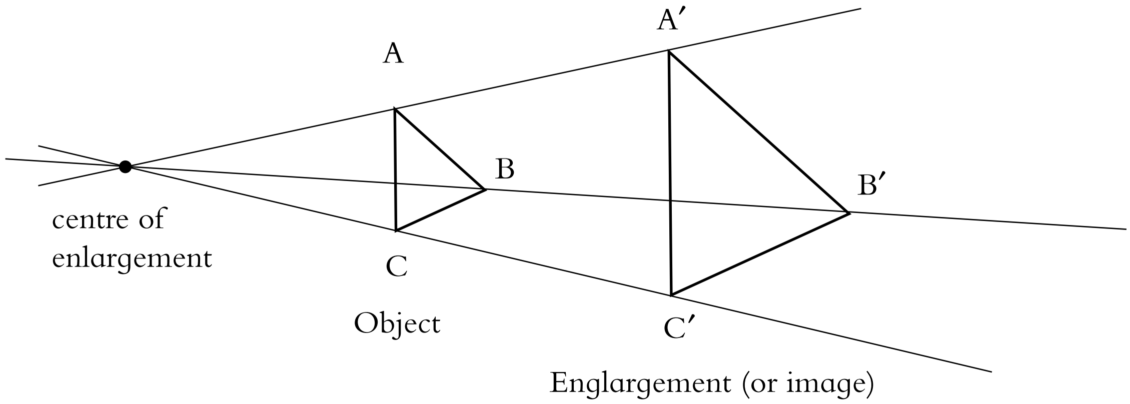


10. Tavita has started to draw an enlargement of the shape below. Copy and complete his enlargement.



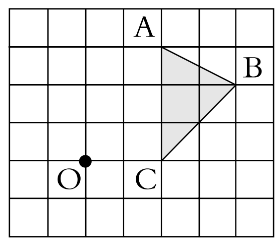
Finding the Centre of Enlargement

The centre of enlargement is the place where lines drawn through corresponding points of an object and its enlargement meet.



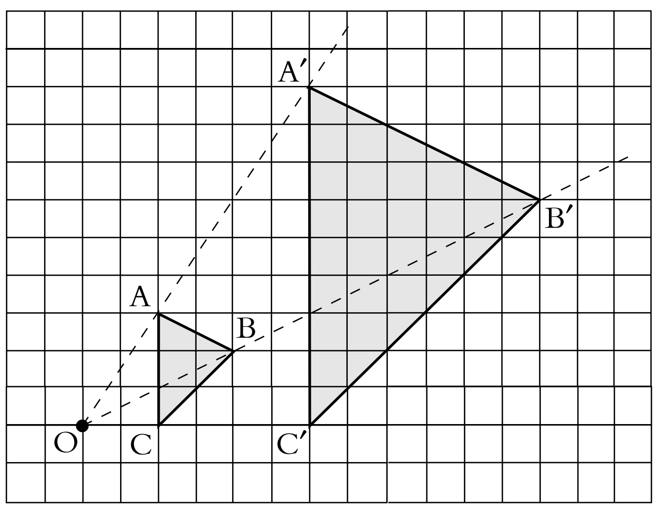
Example 1

The diagram shows the triangle ABC and the point O. Enlarge the triangle with scale factor 3, using O as the centre of enlargement.



Solution

The diagram shows the two triangles. The explanation follows:

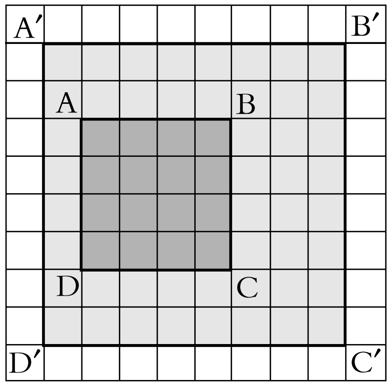


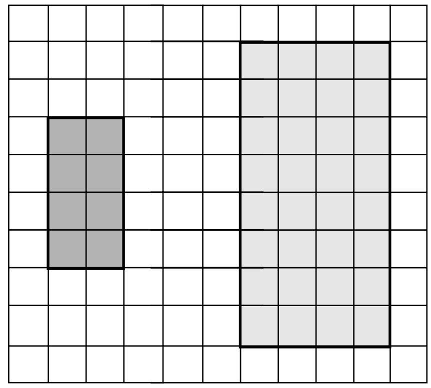
Draw lines from point O through A, B and C, as shown in the diagram. Measure the length O A and multiply it by 3 to get the distance from O of the image point A′, i.e. OA′ = 3 × OA. Mark the point A′ on the diagram. The images B′ and C′ can then be marked in a similar way and the enlarged triangle A′ B′ C′ can then be drawn.

The image will be 3 times bigger than the object.

Example 2

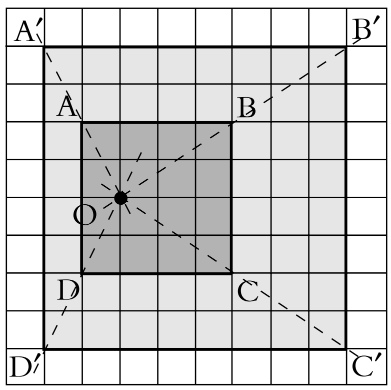
The following diagrams show two shapes that have been enlarged. Find the centre of enlargement in each case.

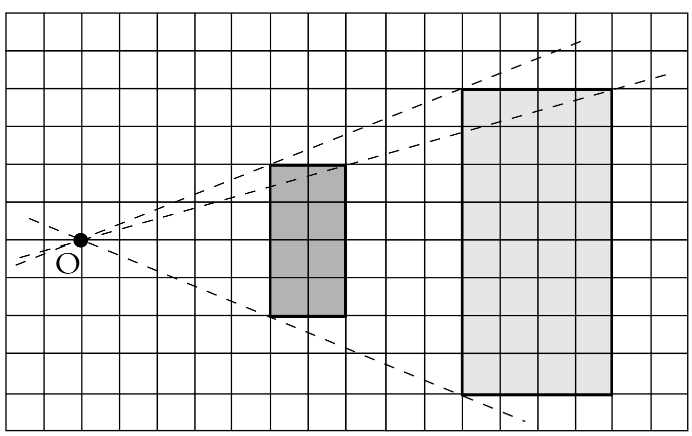




Solution

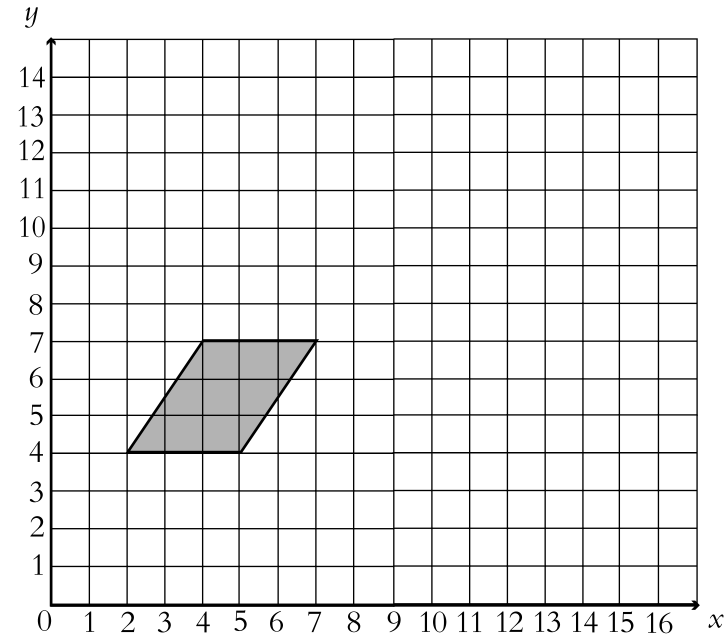
Draw lines through the corresponding corners of each shape. These line will cross at the centre of enlargement. The centres have been marked with the letter O in both diagrams.





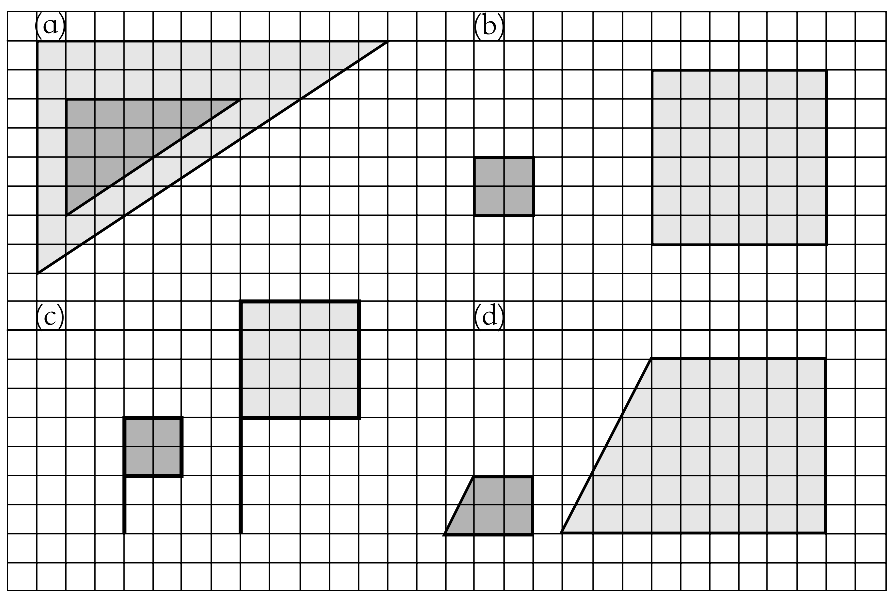
Skill Exercises: Finding the Centre of Enlargement

1. (a) Copy the following diagram:



(b) Enlarge the shape with scale factor 2, using first (0, 0) as the centre of enlargement and then (1, 8) as the centre of enlargement.

2. For each of the following enlargements, copy the diagram and find the centre of enlargement.



3. A triangle has corners at the points with co-ordinates (1, 2), (3, 3) and (0, 3). It is enlarged to give a triangle with corners at the points (5, 4), (11, 7) and (2, 7). Find the scale factor of the enlargement and the co-ordinates of the centre of enlargement.

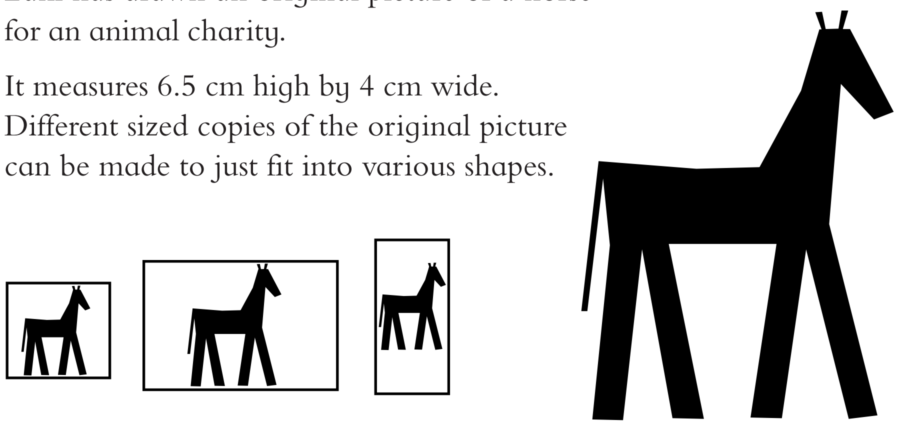
4. A trapezium has corners at the points with co-ordinates (1, 0), (3, 2), (3, 4) and (1, 5). It is enlarged with scale factor 3, using the point (0, 3) as the centre of enlargement.

   Find the co-ordinates of the corners of the enlarged trapezium.

5. A parallelogram has corners at the points with co-ordinates (5, 1), (9, 3), (11, 9), and (7, 7). Enlarge this shape with scale factor $\frac{1}{2}$, using the point with co-ordinates (1, 3) as the centre of enlargement.

6. Lani has drawn an original picture of a horse for an animal charity.

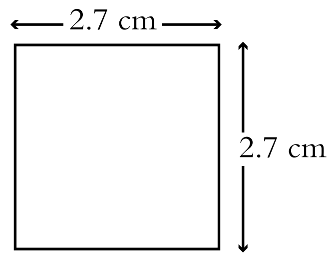
   It measures 6.5 cm high by 4 cm wide. Different sized copies of the original picture can be made to just fit into various shapes.



   (a) Lani wants to enlarge the original picture so that it just fits inside a rectangle on a carrier bag. The rectangle measures 24 cm high by 12 cm wide.

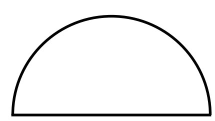
   By what scale factor should she multiply the original picture? Show your working.

   (b) Lani wants to multiply the original picture by a scale factor so that it just fits inside the square shown below for a badge.



   By what scale factor should she multiply the original picture?

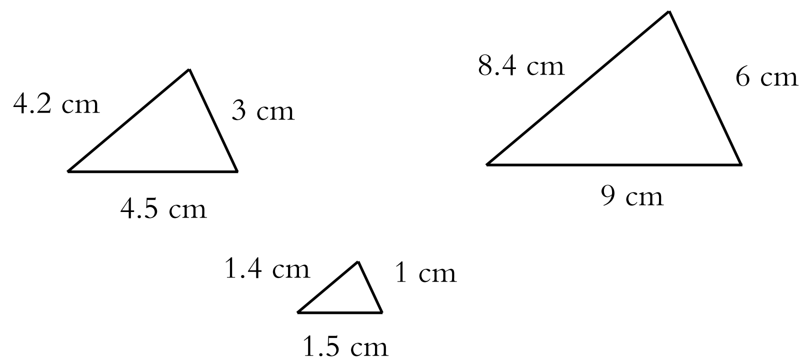
   (c) The original picture is to be used on a poster. It must fit inside a shape like this:



   The shape is to be a semi-circle of radius 6.6 cm. What would be the perimeter of the shape? Show your working.

Recognising Similar Shapes

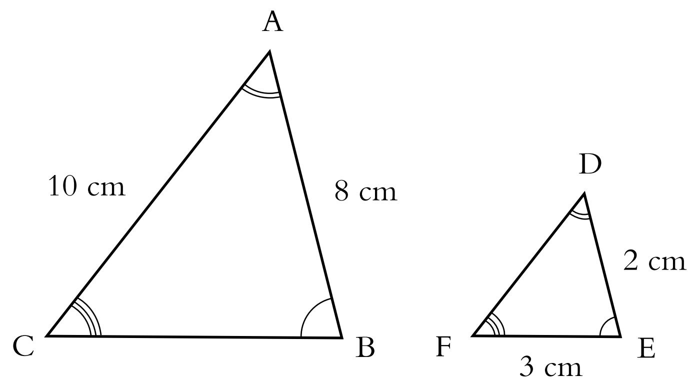
Similar shapes are those which are enlargements of each other. The three triangles shown below are similar:



It is possible to calculate the lengths of the sides of similar shapes.

Example 1

The following diagram shows two similar triangles:



Calculate the lengths of the sides BC and DF.

Solution

Comparing the sides AB and DE gives

$$AB = 4 \times DE$$

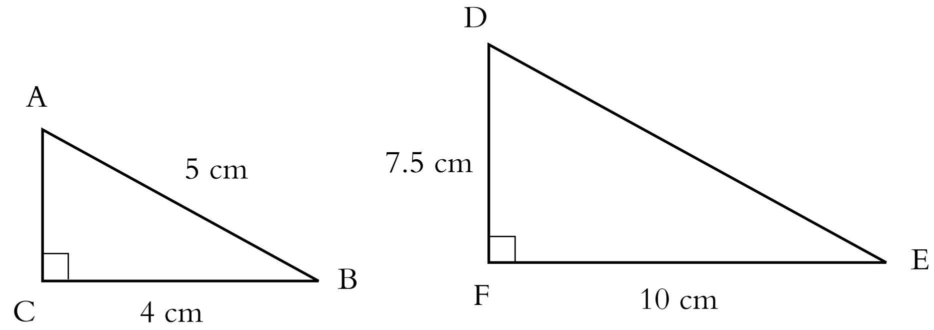
Therefore, all the lengths in the triangle ABC will be four times the lengths of the sides in the triangle DEF.

$$\begin{aligned} BC &= 4 \times EF \\ &= 4 \times 3 \\ &= 12 \text{ cm} \end{aligned}$$

$$\begin{aligned} AC &= 4 \times DF \\ 10 &= 4 \times DF \\ DF &= \frac{10}{4} \\ &= 2.5 \text{ cm} \end{aligned}$$

Example 2

The following diagram shows two similar triangles:



Calculate the lengths of the sides AC and DE.

Solution

Comparing the lengths BC and EF gives

EF $=$ $2.5 \times$ BC

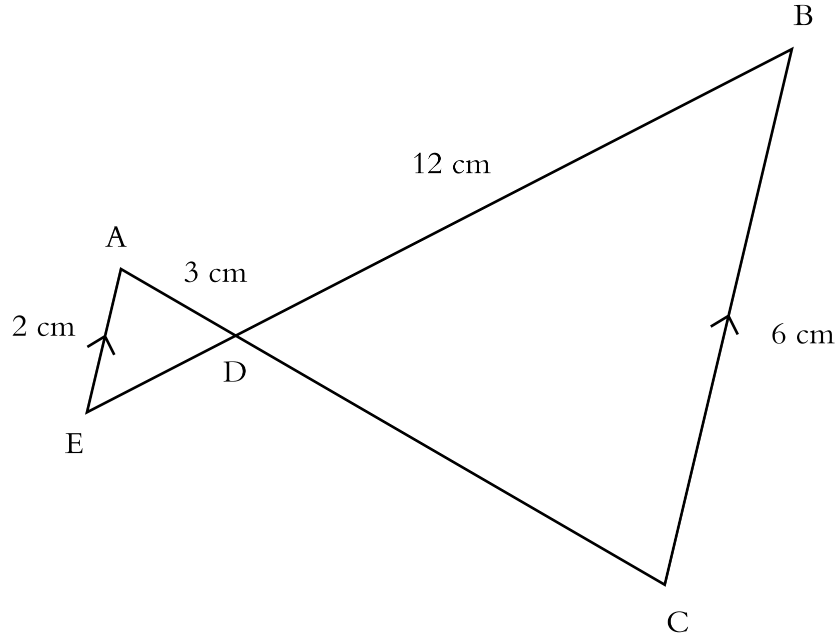
Therefore, the lengths in the triangle DEF are 2.5 times longer than the lengths in the triangle ABC.

$$\begin{aligned} DE &= 2.5 \times AB \\ &= 2.5 \times 5 \\ &= 12.5 \text{ cm} \end{aligned}$$

$$\begin{aligned} DF &= 2.5 \times AC \\ 7.5 &= 2.5 \times AC \\ AC &= \frac{7.5}{2.5} \\ &= 3 \text{ cm} \end{aligned}$$

Example 3

In the following diagram, the sides AE and BC are parallel:



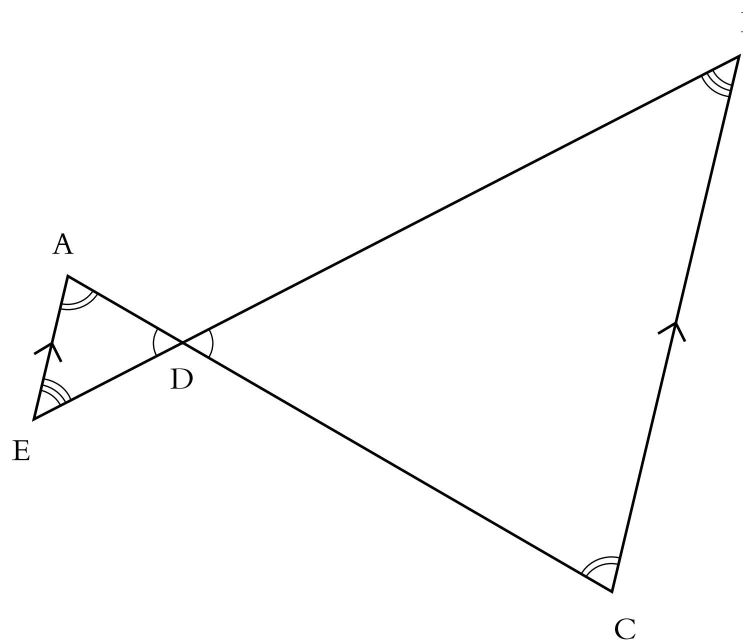
(a) Explain why ADE and CDB are similar triangles.

(b) Calculate the lengths DE and CD.

Solution

(a) $\angle ADE$ and $\angle CDB$ are opposite angles and so are equal.

Because AE and BC are parallel, $\angle DBC = \angle DEA$ and $\angle EAD = \angle BCD$. They are alternate angles.



As the triangles have angles the same size, they must be similar.

(b) Comparing AE and BC shows that the lengths in the larger triangle are three times the lengths of the sides in the smaller triangle, so

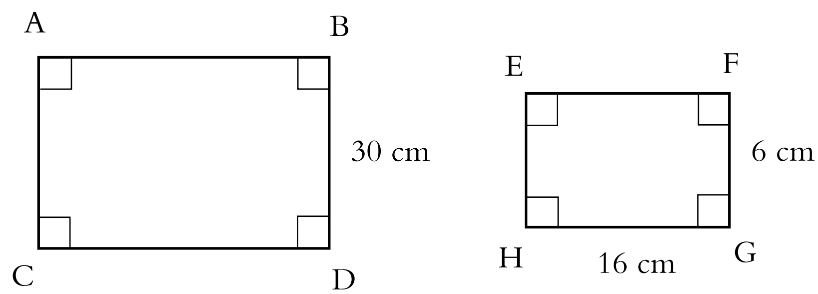
$$\begin{aligned} DC &= 3 \times AD \\ &= 3 \times 3 \\ &= 9 \text{ cm} \end{aligned}$$

and

$$\begin{aligned} BD &= 3 \times DE \\ 12 &= 3 \times DE \\ DE &= \frac{12}{3} \\ &= 4 \text{ cm} \end{aligned}$$

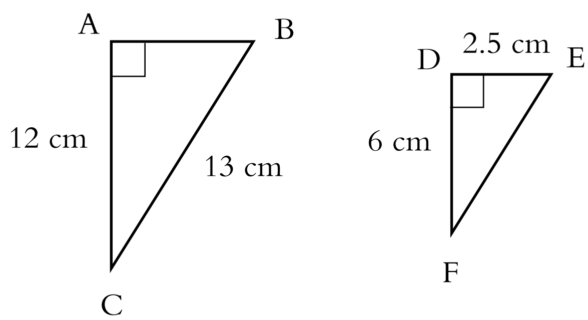
Skill Exercises: Recognising Similar Shapes

1. The following diagram shows two similar rectangles:



Determine the length of the side CD.

2. The following diagram shows two similar triangles:

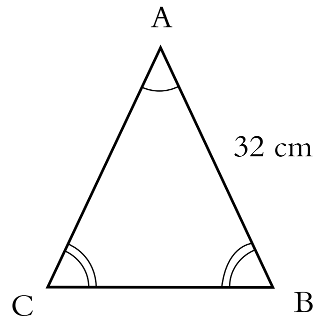


Calculate the lengths of:

(a) AB

(b) EF

3. Two similar isosceles triangles are shown in the diagram below:

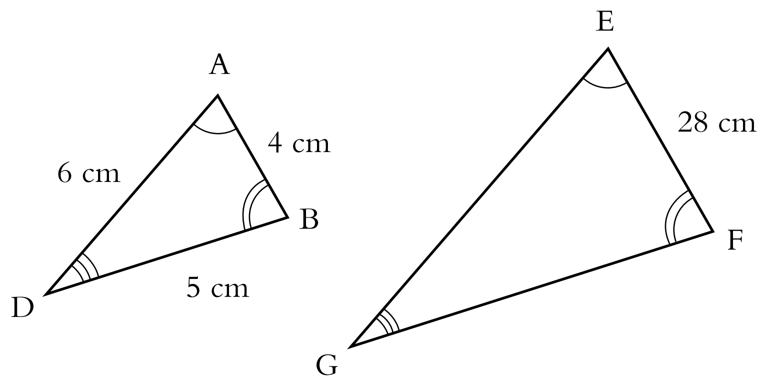


(a) What is the length of DE?

(b) What is the length of AC?

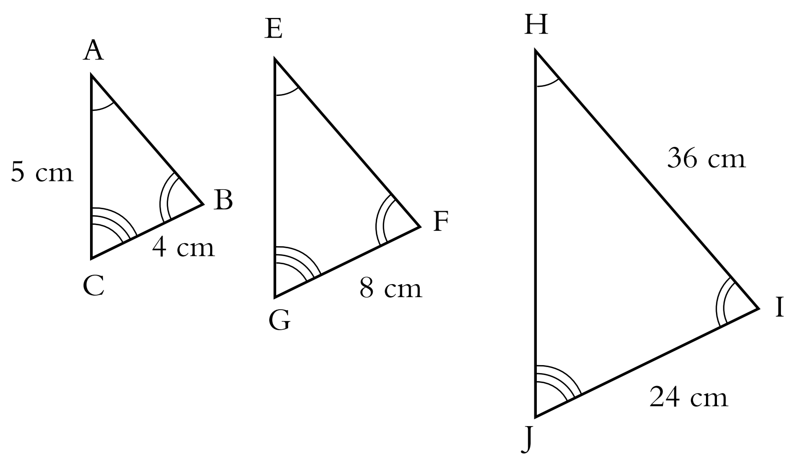
(c) Calculate the length of BC.

4. The following diagram shows two similar triangles:



Calculate the lengths of the sides GE and FG.

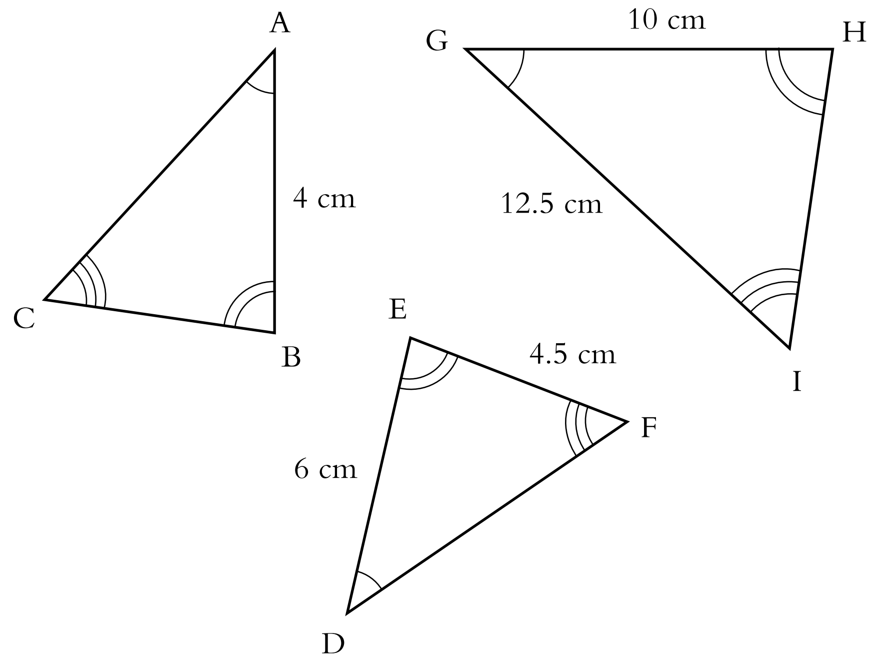
5. The following diagram shows three similar triangles:



Calculate the length of:

(a) EG (b) HJ (c) EF (d) AB

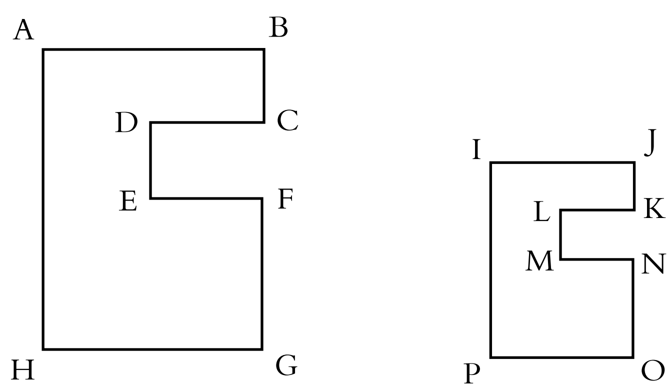
6. The following diagram shows three similar triangles:



Calculate the length of the sides:

(a) HI (b) BC (c) AC (d) DF

7. The following diagram shows two similar shapes:



The length of the side AB is 6 cm and the length of the side IJ is 4 cm.

(a) If AH = 12 cm, calculate the length IP.

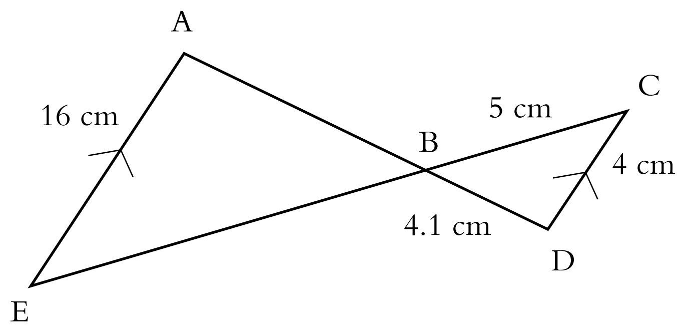
(b) If BC = 3 cm, calculate the length JK.

(c) If DE = BC, determine the length LM.

(d) Calculate the lengths FG and NO.

(e) If MN = 3 cm, determine the length EF.

8. In the diagram below, the lines AE and CD are parallel.



(a) Copy and complete the following statements:

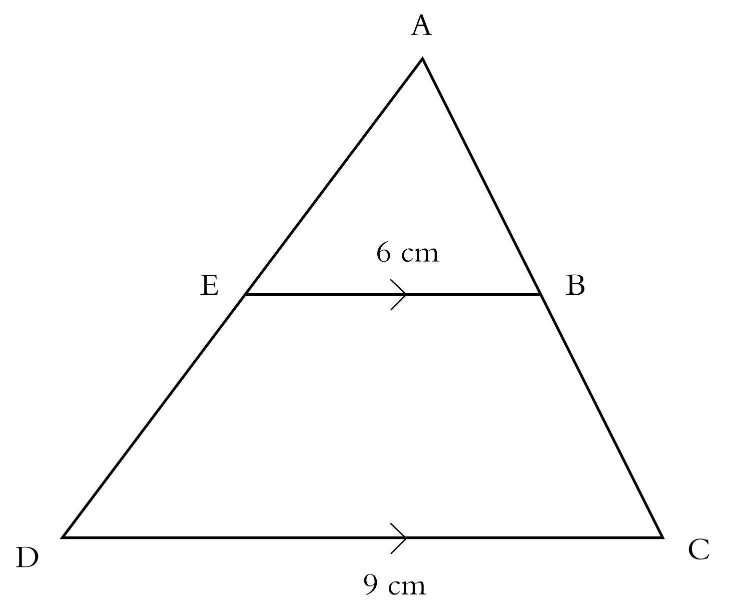
$\angle ABE = \angle$

$\angle BAE = \angle$

$\angle AEB = \angle$

(b) Calculate the lengths of AB and BE.

9. In the diagram shown below the lines EB and DC are parallel.



(a) Explain why the triangles ABE and ACD are similar.

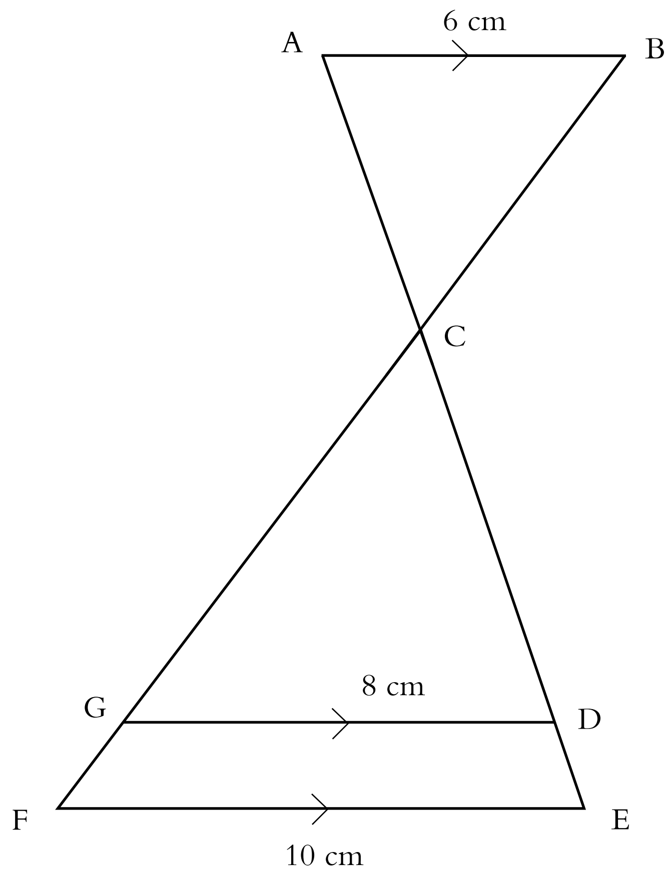
(b) If the length of AB is 4.4 cm, calculate the lengths of AC and BC.

(c) If the length of AD is 13.5 cm, determine the lengths of AE and DE.

10. In the diagram shown, the lines AB, GD and FE are parallel.

    (a) If the length of CE is 15 cm, calculate the lengths of AC, CD and DE.

    (b) If the length of BC is 10.8 cm, calculate the length of FG.



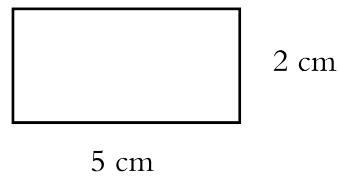
Working with Line, Area and Volume Ratios

In this section we consider what happens to the area and volume of shapes when they are enlarged.

Example 1

The rectangle shown is enlarged with scale factor 2 and scale factor 3.

What happens to the area for each scale factor?



Solution

The area of the original rectangle is

area $= 5 \times 2$

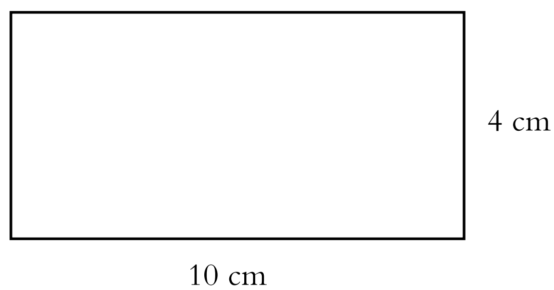
$= 10 \text{ cm}^2$

For an enlargement scale factor 2, the rectangle becomes:

area $= 10 \times 4$

$= 40 \text{ cm}^2$

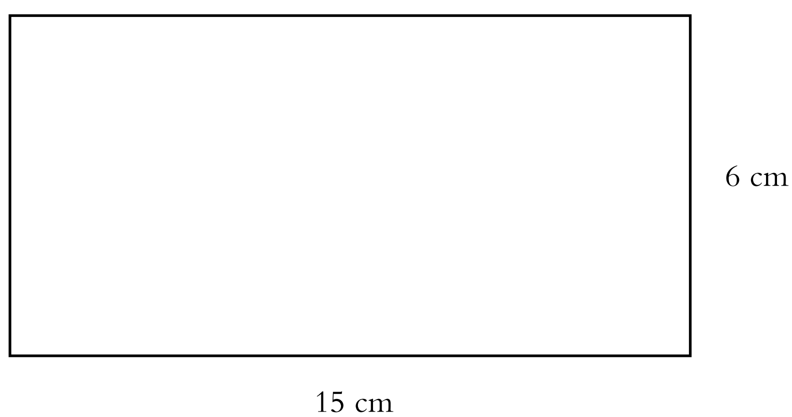
The area has increased by a factor of 4, or 2^2.



For an enlargement scale factor 3, the rectangle becomes:

area $= 15 \times 6$

$= 90 \text{ cm}^2$



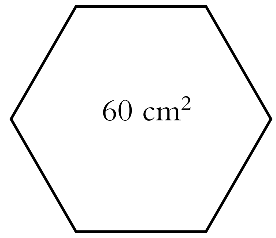
The area has increased by a factor of 9, or 3^2.

If a shape is enlarged with scale factor k, its area is increased by a factor k^2.

Example 2

A hexagon has area 60 cm^2.

Calculate the area of the hexagon, if it is enlarged with scale factor:



(a) 2 (b) 4 (c) 10

Solution

In each case the area will increase by the scale factor squared.

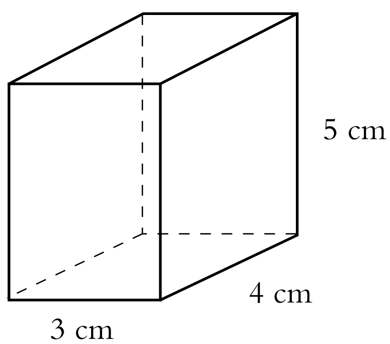
(a) New area $= 2^2 \times 60$
$= 4 \times 60$
$= 240$ cm^2

(b) New area $= 4^2 \times 60$
$= 16 \times 60$
$= 960$ cm^2

(c) New area $= 10^2 \times 60$
$= 100 \times 60$
$= 6000$ cm^2

Example 3

A cuboid has sides of lengths 3 cm, 4 cm and 5 cm.



Calculate the volume of the cuboid, if it is enlarged with scale factor:

(a) 2

(b) 10

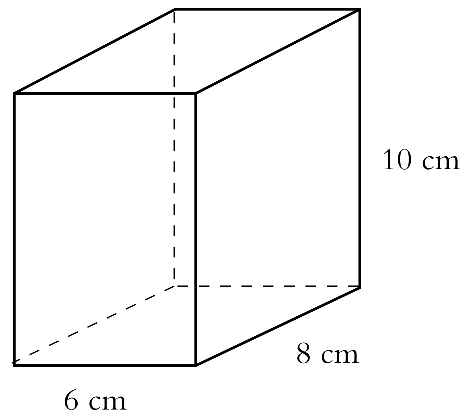
Solution

(a) The dimensions of the cuboid now become,

6 cm, 8 cm and 10 cm

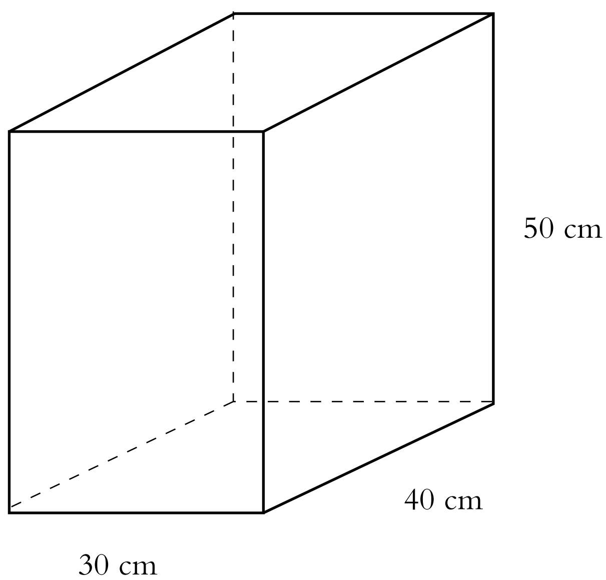
New volume $= 6 \times 8 \times 10$

$= 480 \text{ cm}^3$



The volume of the original cuboid was 60 cm^3. The volume has increased by a factor of 8, or 2^3.

(b) The dimensions of the cuboid now become 30 cm, 40 cm and 50 cm.



New volume $= 30 \times 40 \times 50$

$= 60\ 000 \text{ cm}^3$

This is 1000, or 10^3, times bigger than the volume of the original cuboid.

If a solid is enlarged with scale factor k, its volume is increased by a factor k^3.

Example 4

A sphere has a volume of 20 cm^3. A second sphere has 4 times the radius of the first sphere. Calculate the volume of the second sphere.

Solution

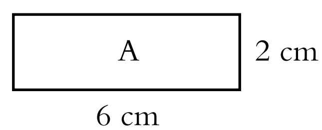
The radius is increased by a factor of 4.

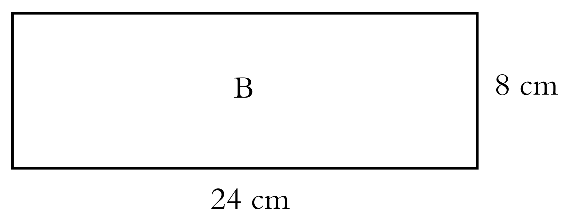
The volume will be increased by a factor of 4^3.

$$\begin{aligned}\text{Volume} &= 20 \times 4^3\\ &= 20 \times 64\\ &= 1280 \text{ cm}^3\end{aligned}$$

Skill Exercises: Working with Line, Area and Volume Ratios

1. Two rectangles are shown below:





   (a) Calculate the area of each rectangle.

   (b) What is the scale factor of enlargement between rectangle A and rectangle B?

   (c) How is the area of rectangle B affected when compared to the area of rectangle A?

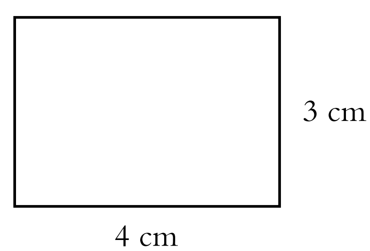
2. Calculate the area of the rectangle shown if it is enlarged with a scale factor of:

   (a) 2

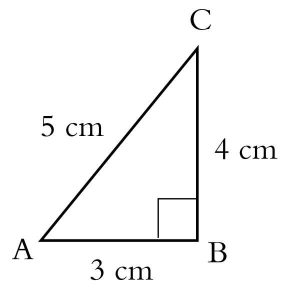
   (b) 3

   (c) 6

   (d) 10



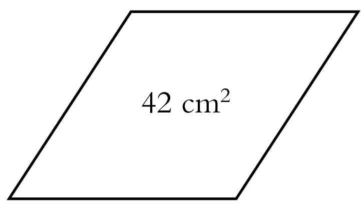
3. The following table gives information about enlargements of the triangle shown, which has an area of 6 cm^2.



Copy and complete the table:

Length of Sides Base	Height	Scale Factor	Area	Area Scale Factor
3 cm	4cm	1	6 cm^2	1
		2		
	12 cm			
	16 cm			
15 cm				
		6		
30 cm	40 cm		600 cm^2	100
4.5 cm				

4. The parallelogram shown has an area of 42 cm^2.



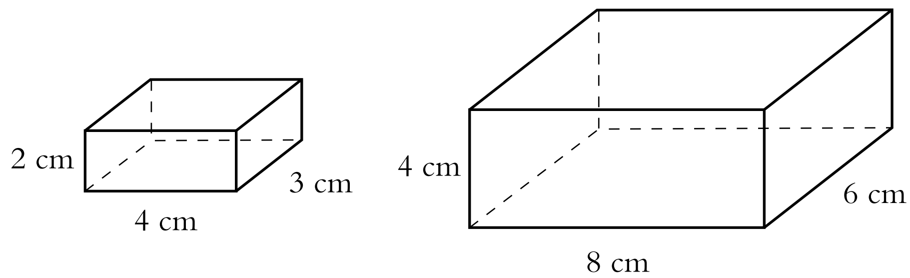
The parallelogram is enlarged with a scale factor of 5.

Calculate the area of the enlarged parallelogram.

5. The area of a circle is 50 cm^2. A second circle has a radius that is three times the radius of the first circle. What is the area of this circle?

6. Two similar rectangles have areas of 30 cm^2 and 480 cm^2. Describe how the length and width of the two rectangles compare.

7. (a) Find the volume of each of the following cuboids:



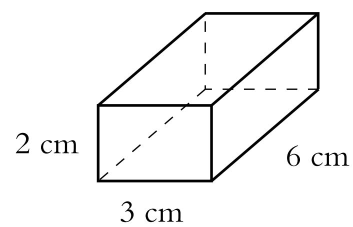
(b) The larger cuboid is an enlargement of the smaller cuboid. What is the scale factor of the enlargement?

(c) How many of the smaller cuboids can be fitted into the larger cuboid?

(d) How many times greater is the volume of the larger cuboid than the volume of the smaller cuboid?

8. A cuboid has dimensions as shown in the diagram.

The cuboid is enlarged to give larger cuboids. Copy and complete the following table:



Dimensions			Scale Factor	Volume	Volume Factor
Width	Length	Height			
3 cm	6 cm	2 cm	1	36 cm^3	1
6 cm			2		
			4		
		10 cm			
30 cm					

9. A tank has a volume of 32 m^3. It is enlarged with scale factor 3. What is the volume of the enlarged tank?

10. A cylinder has height 10 cm and volume 42 cm^3. An enlargement of the cylinder has height 25 cm. Calculate the volume of the enlarged cylinder.

Using Enlargement with Maps and Scale Models

Areas and Volumes change after an enlargement. This concept should be applied when using maps or building scale models.

If a map has a scale $1 : n$, then:
lengths have a scale of $1 : n$
and areas have a scale of $1 : n^2$

If a model has a scale of $1 : n$, then
lengths have a scale of $1 : n$
areas have a scale of $1 : n^2$
and volumes have a scale of $1 : n^3$

Note on units		
1 km	=	1000 m
	=	100 000 cm
1 m^2	=	10 000 cm^2
1 km^2	=	1 000 000 m^2
1 m^3	=	1 000 000 cm^3

Example 1

On a map with a scale of 1 : 20 000, a plantation has an area of 5 cm^2. Calculate the actual area of the plantation.

Solution

Actual area $= 5 \times 20\,000^2$
$= 2\,000\,000\,000$ cm^2
$= 200\,000$ m^2 (dividing by 10 000)
$= 0.2$ km^2 (dividing 1 000 000)

Example 2

A map has a scale of 1 : 500. A small garden on the map has an area of 14 cm^2. Calculate the actual area of this garden.

Solution

Actual area $= 14 \times 500^2$
$= 3\,500\,000$ cm^2
$= 350$ m^2

Example 3

A model car is made on a scale of 1 : 20.

The length of the model is 24 cm.

The area of the windscreen of the model is 32 cm^2.

The volume of the boot of the model is 90 cm^3.

Calculate the actual:

(a) length of the car

(b) area of the windscreen

(c) volume of the boot

Solution

(a) Actual length of car $= 24 \times 20$

$= 480$ cm

$= 4.8$ m

(b) Actual area of windscreen $= 32 \times 20^2$

$= 12\ 800\ cm^2$

$= 1.28\ m^2$

(c) Actual volume of boot $= 90 \times 20^3$

$= 720\ 000\ cm^3$

$= 0.72\ m^3$

Skill Exercises: Using Enlargement with Maps and Scale Models

1. A model boat is made to a scale of 1 : 10.

   The length of the model is 40 cm.

   The area of the hull of the model is 500 cm^2.

   The volume of the hull of the model is 3200 cm^3.

   Calculate the actual:

   (a) length of the boat

   (b) area of the hull

   (c) volume of the hull

2. A map has a scale of 1 : 50 000. On the map the area of a lake is 50 cm^2. Calculate the actual area of the lake in:

   (a) cm^2 (b) m^2 (c) km^2

3. A model of a building is made with a scale of 1 : 60. The volume of the model is 36 000 cm^3. Calculate the volume of the actual building in m^3.

4. A piece of land is represented on a map by a rectangle 2 cm by 5 cm. The scale of the map is 1 : 40 000. Calculate the area of the piece of land in:
   (a) cm^2
   (b) m^2
   (c) km^2

5. A model of a house is made to a scale of 1 : 30.

   The height of the model is 20 cm.

   The area of the roof of the model is 850 cm^2.

   The volume of the model house is 144 400 cm^3.

   Calculate the actual:
   (a) height of the house in m
   (b) area of the roof in m^2
   (c) volume of the house in m^3

6. An aeroplane has a wingspan of 12 m. A model of this plane has a wingspan of 60 cm.
   (a) Calculate the scale of the model.
   (b) The volume of the model is 3015 cm^3. Calculate the volume of the actual aeroplane in m^3.
   (c) A badge on the model has area 2 cm^2. Calculate the area of the actual badge in cm^2 and m^2.

7. A forest has an area of 4 cm^2 on a map with a scale of 1 : 200 000. Calculate the actual area of the forest in km^2.

8. An island has an area of 50 km^2. What would be the area of the island on a map with a scale of 1 : 40 000?

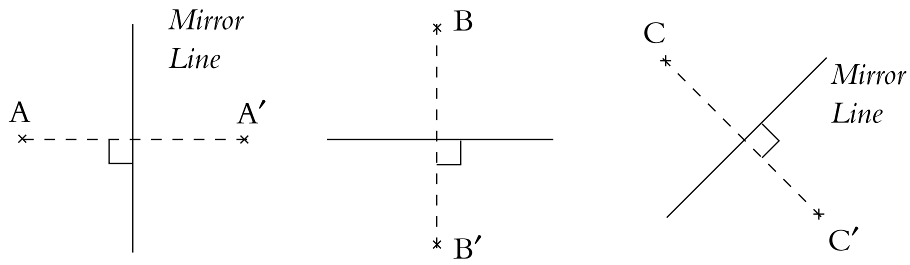
9. An indoor sports stadium has 5000 seats surrounding a playing area with an area of 384 m^2. The total volume of the stadium is 3840 m^3. A model is made to a scale of 1 : 80.
   (a) How many seats are there in the model?
   (b) What is the area of the playing surface in the model in cm^2?
   (c) What is the volume of the model in cm^3?

10. A forest has an area of 5 km^2. On a map it is represented by an area of 20 cm^2. What is the scale of the map?

Section 7.2 Exploring Pattern Using Line And Rotational Symmetry

Working with Line Symmetry and Reflection

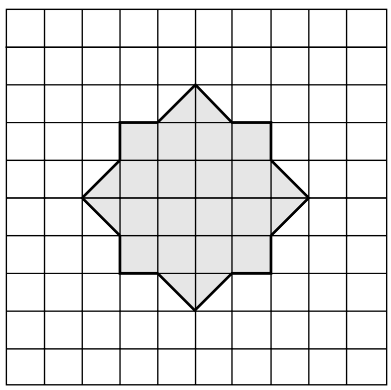
In a reflection, a point will move to a new position that will be the same distance from the mirror line, but on the other side. These distances will always be measured at right angles to the mirror line. A line of symmetry divides a shape into two or more parts of the same size and shape.



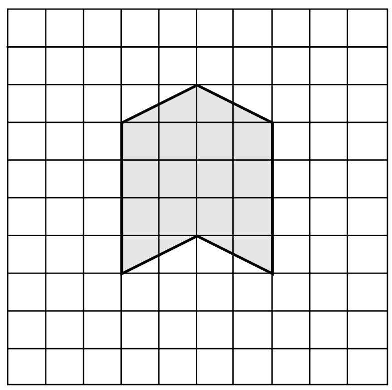
Example 1

Draw in the lines of symmetry for each of the following shapes:

(a)

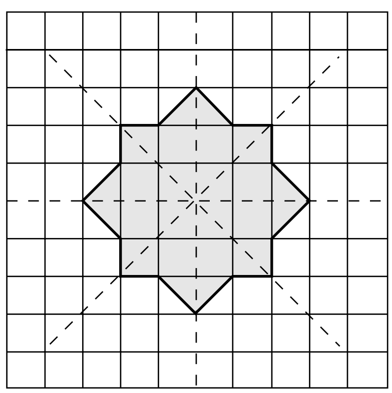


(b)



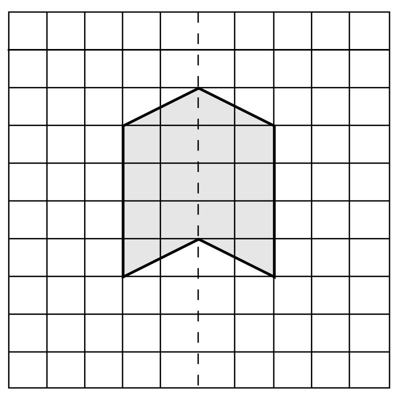
Solution

(a)



4 lines of symmetry

(b)

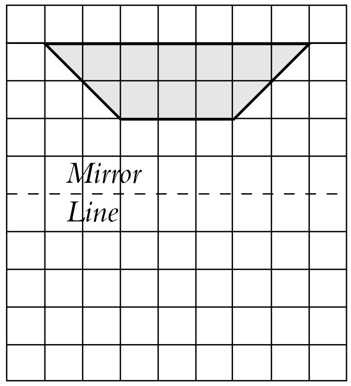


1 line of symmetry

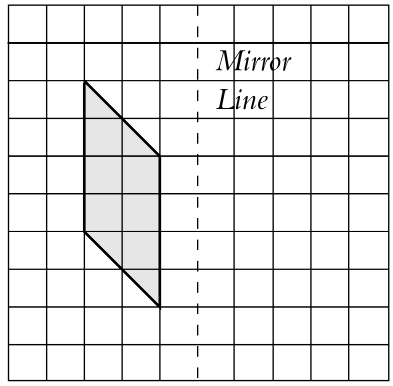
Example 2

Draw the reflection of each shape in the given mirror line:

(a)

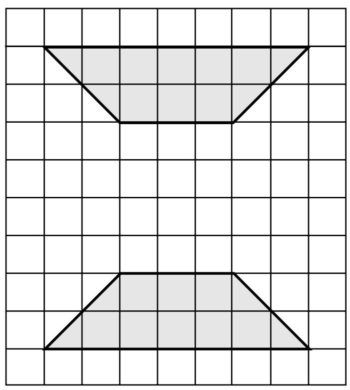


(b)

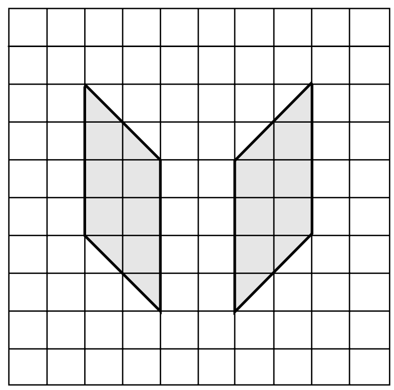


Solution

(a)



(b)



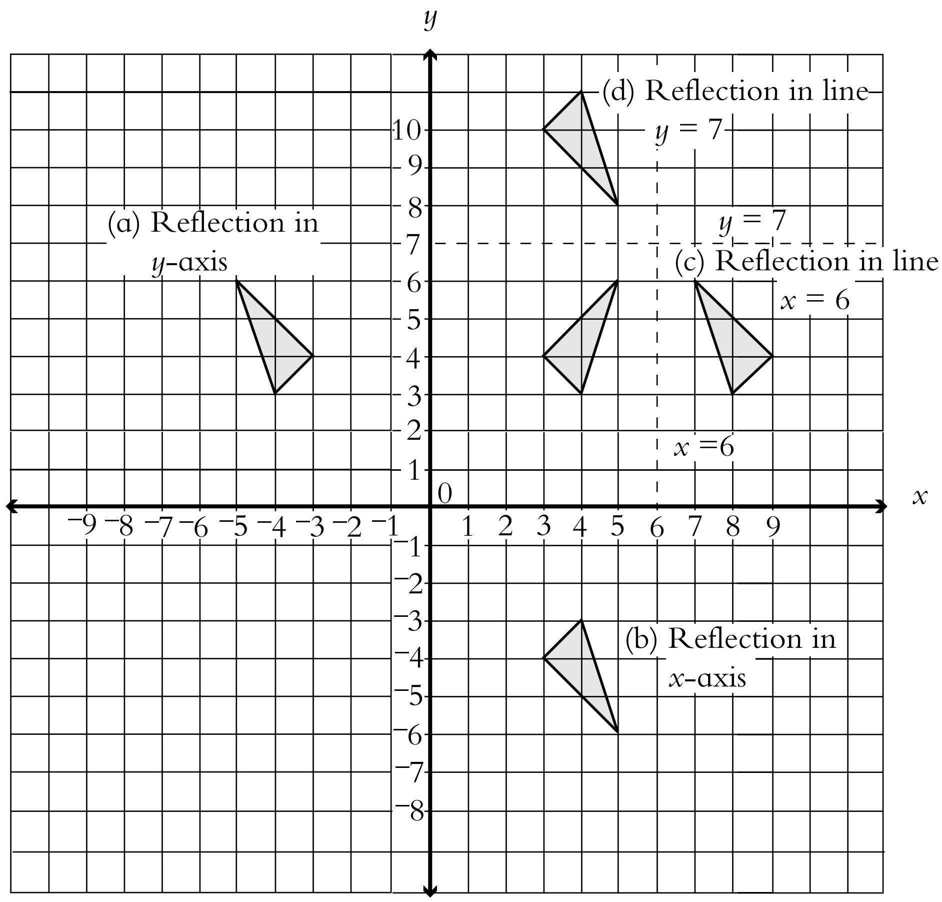
Example 3

A triangle has corners at the points with co-ordinates $(4, 3), (5, 6)$ and $(3, 4)$. Draw the reflection of the triangle in the:

(a) x-axis (b) y-axis

(c) line $x = 6$ (d) line $y = 7$

Solution



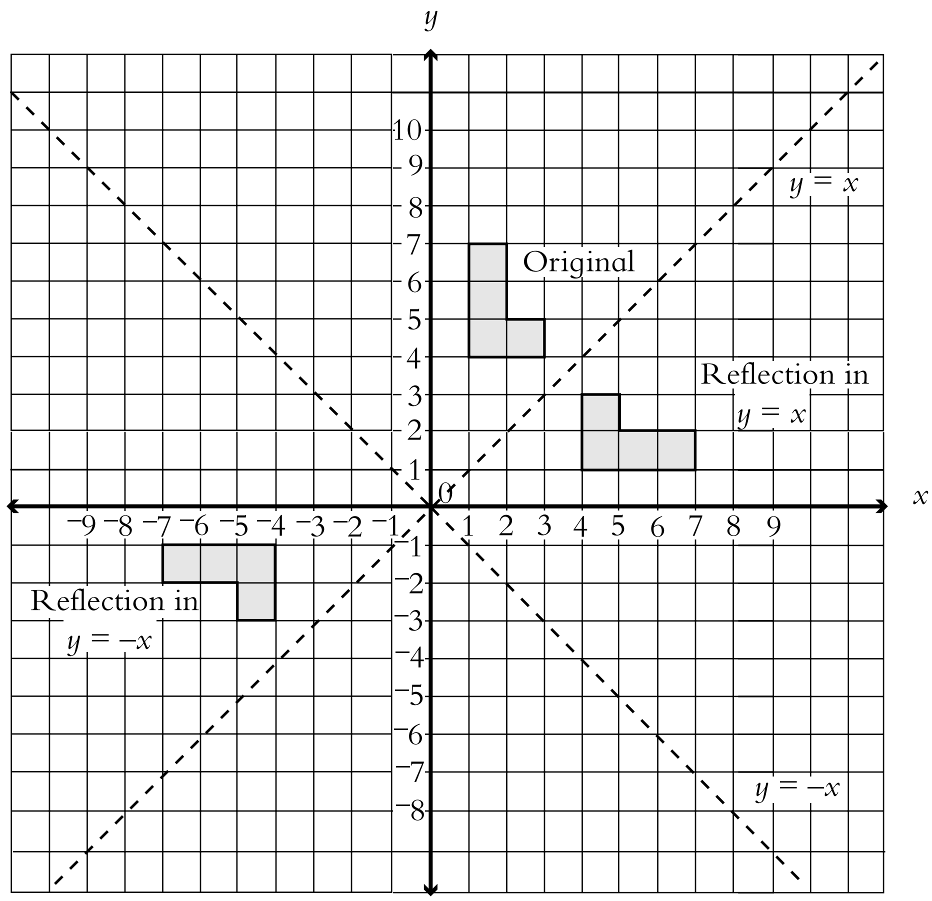
Example 4

An 'L' shape has corners at the points with co-ordinates (1, 4), (1, 7), (2, 7), (2, 5), (3, 5) and (3, 4).

Draw the reflection of the shape in the lines:

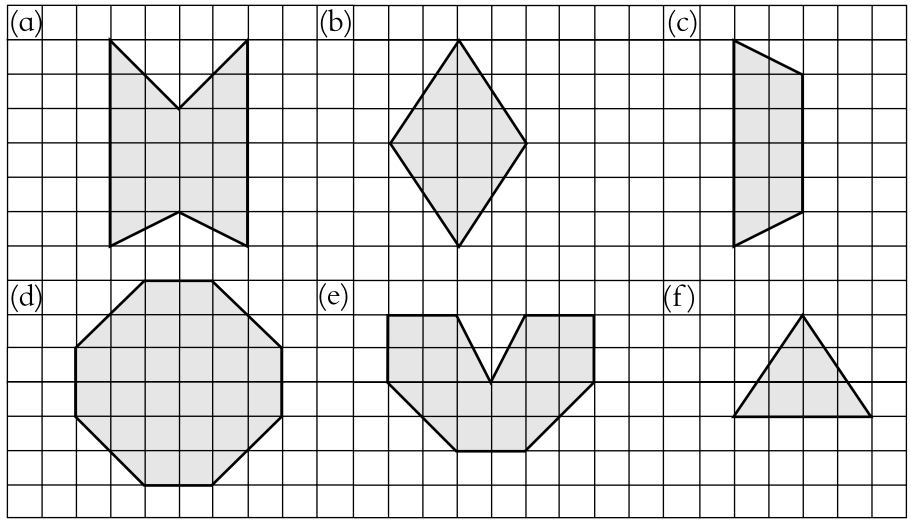
(a) $y = x$ (b) $y = {}^{-}x$

Solution

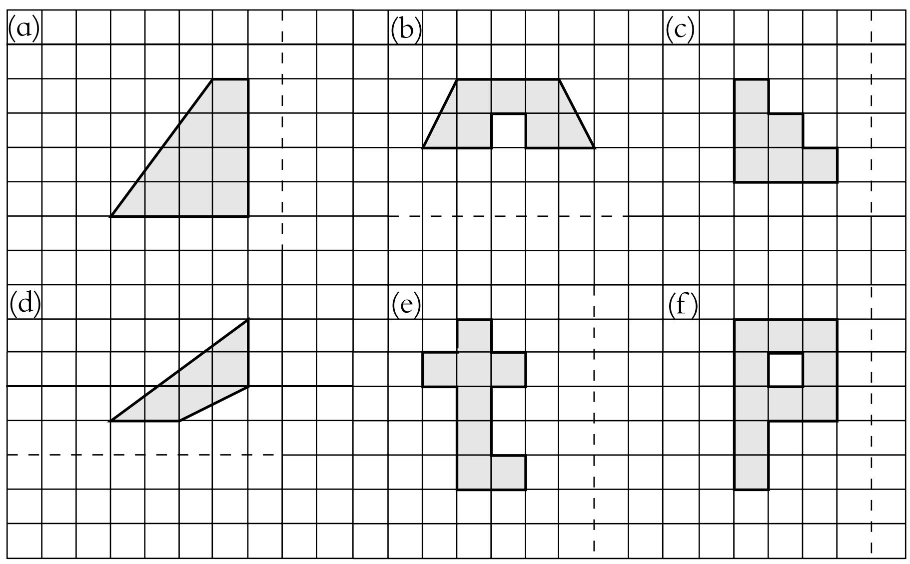


Skill Exercises: Working with Line Symmetry and Reflection

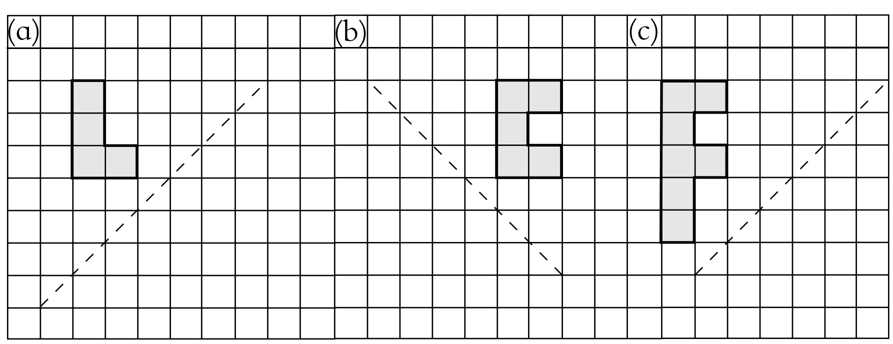
1. Copy the following shapes and draw in all their lines of symmetry:



2. Draw the reflection of each of the following shapes in the line given:

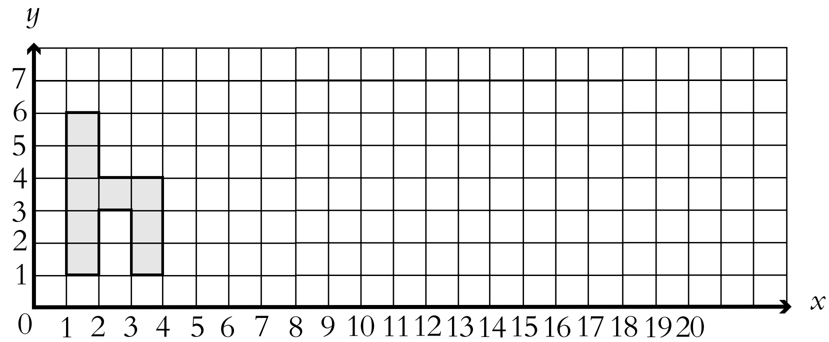


3. Copy each of the following shapes and draw its reflection in the line shown:

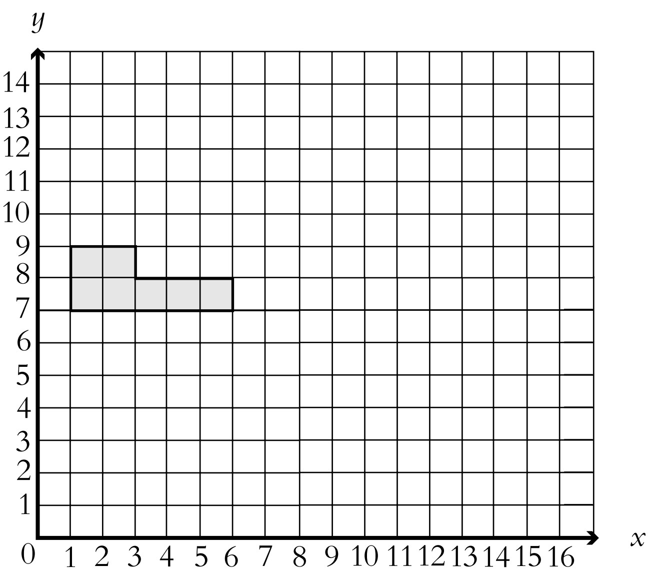


4. (a) Copy the following diagram:

   (b) Reflect the shape in the lines $x = 8$ and $x = 11$.



5. (a) Copy the diagram shown:



   (b) Reflect the shape in the lines $y = 10$, $y = 5$ and $x = 7$.

6. (a) Draw the triangle that has corners at the points with co-ordinates (1, 1), (4, 7) and (2, 5).

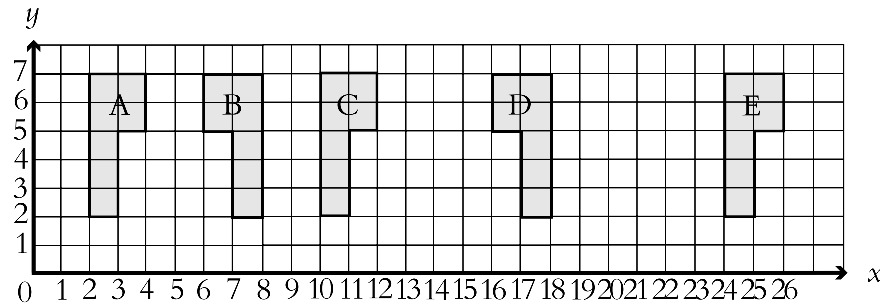
   (b) Reflect the triangle in the lines:

      (i) $x = 8$

      (ii) $x = -1$

      (iii) $y = -2$

7. The following diagram shows the shapes A, B, C, D and E.



Write down the equation of the mirror line for each of the following reflections:

(a) A to B (b) B to C (c) A to D

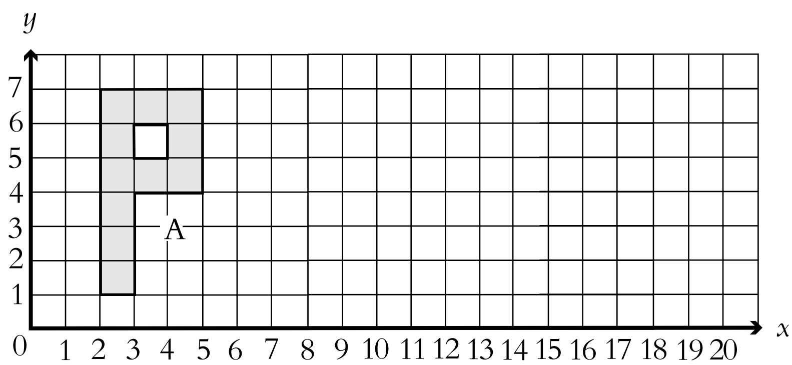
(d) B to E (e) D to E (f) C to D

8. (a) Draw the triangle which has corners at the points with co-ordinates (1, 4), (1, 7) and (3, 5).

(b) Reflect this shape in the line $y = x$ and state the co-ordinates of the corners of the reflected shape.

(c) Reflect the original triangle in the line $y = {}^{-}x$ and state the co-ordinates of the corners of the reflected shape.

9. (a) Draw the shape A shown in the following diagram.



(b) Reflect the shape A in the line $x = 6$ to obtain shape B.

(c) Reflect the shape B in the line $x = 14$ to obtain shape C.

(d) Describe the translation that would take shape A straight to shape C.

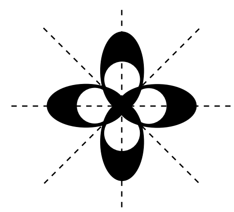
10. Draw the triangle with corners at the points with co-ordinates (1, 3), (1, 8) and (6, 8). Reflect this triangle in the following lines:

    (a) $x = 0$ (b) $y = 0$ (c) $y = x$ (d) $y = {}^{-}x$

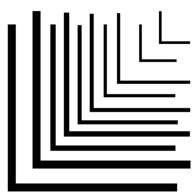
11. These patterns have one or more lines of symmetry. Draw all the lines of symmetry in each pattern.

    You may use a mirror or tracing paper to help you.

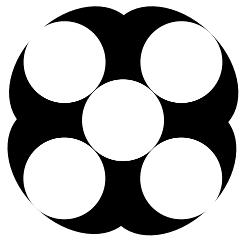
Example



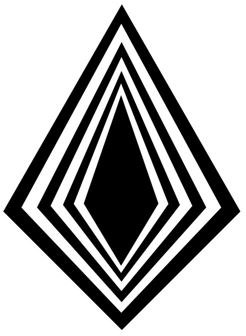
(a)



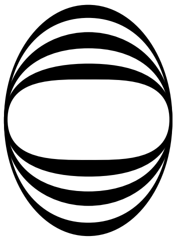
(b)



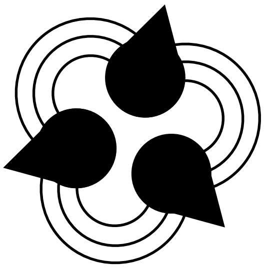
(c)



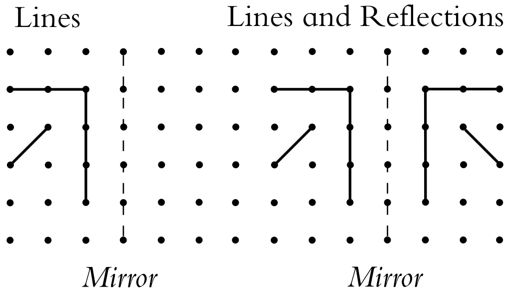
(d)



(e)



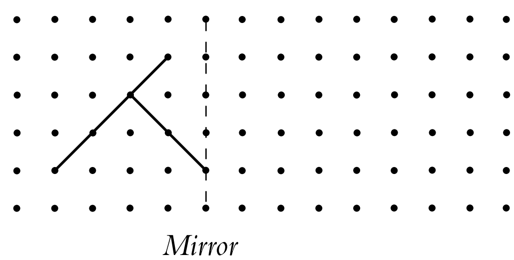
12. Nina is making patterns. To make a pattern she drawn some lines on a grid. The she reflects them in a mirror line.



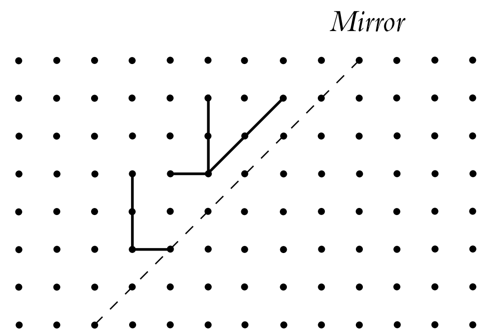
Make a copy of each of the following grids and lines.

Reflect each group of lines in its mirror line to make a pattern. You may use a real mirror or tracing paper to help you.

(a)



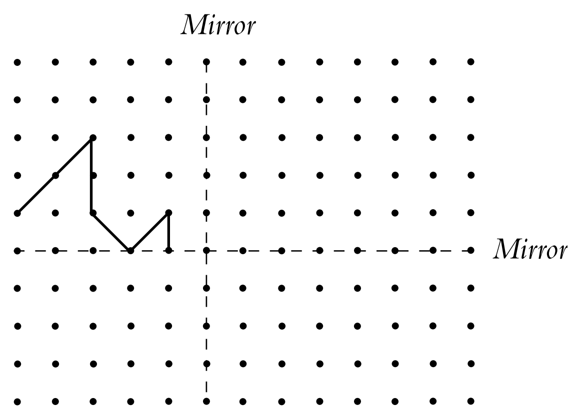
(b)



(c) Now use two mirror lines to make a pattern.

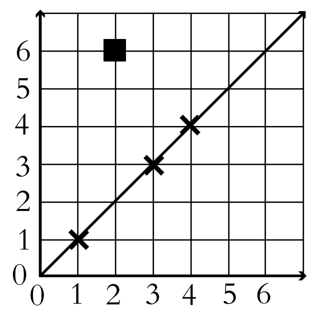
First reflect the group of lines in one mirror line to make a pattern.

Then reflect the whole pattern in the other mirror line.



13. (a) Three points on this line are marked with ✕.

Their co-ordinates are (1, 1), (3, 3) and (4, 4). Look at the numbers in the co-ordinates of each point. What do you notice?



(b) The point $(?, 14\frac{1}{2})$ is on the line. Write down its missing co-ordinate.

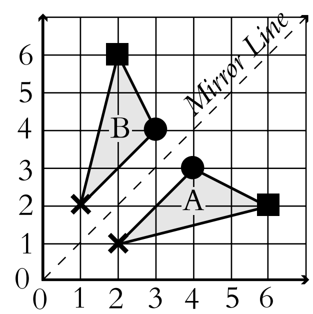
(c) The point ■ is above the line.

Four points are at (10, 10), (10, 12), (12, 10) and (12, 12). Which one of these points is above the line? Explain why.

(d) The point (?, 15) is above the line. Write down a possible co-ordinate for the point.

(e) Look at triangles A and B.

	Triangle A	Triangle B
Co-ordinates of ●	(4, 3)	(3, 4)
Co-ordinates of ✕	(2, 1)	(1, 2)
Co-ordinates of ■	(6, 2)	(2, 6)

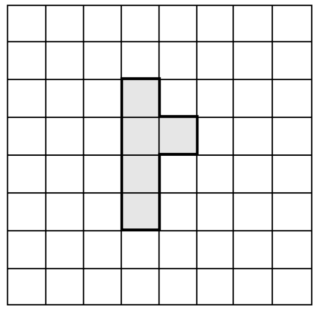


Triangle A was reflected onto triangle B.

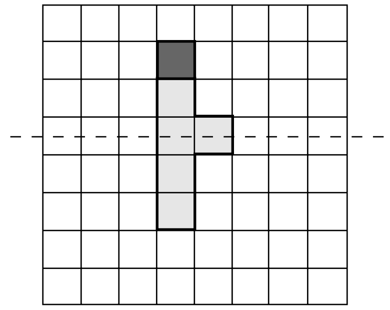
What happened to the numbers in the co-ordinates of each corner?

(f) Alena wants to reflect the point (20, 13) in the mirror line. What point will (20, 13) go to?

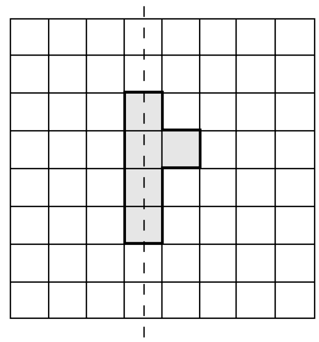
14. Lina shades in a shape made of five squares on a grid:



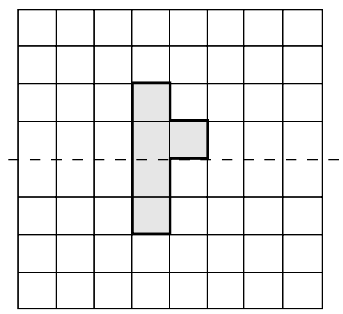
She shades in one more square to make a shape which has the dashed line as a line of symmetry.



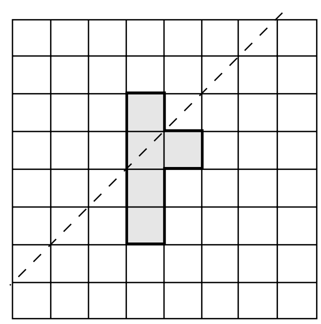
(a) On a copy of the grid opposite, shade in one more square to make a shape which has the dashed line as a line of symmetry.



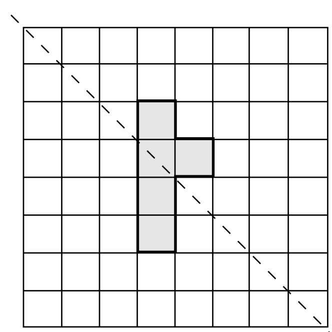
(b) On a copy of the grid opposite shade in one more square to make a shape which has the dashed line as a line of symmetry. You may use a mirror or tracing paper to help you.



(c) On a copy of the grid opposite, shade in two more squares to make shape which has the dashed line as a line of symmetry. You may use a mirror or tracing paper to help you.



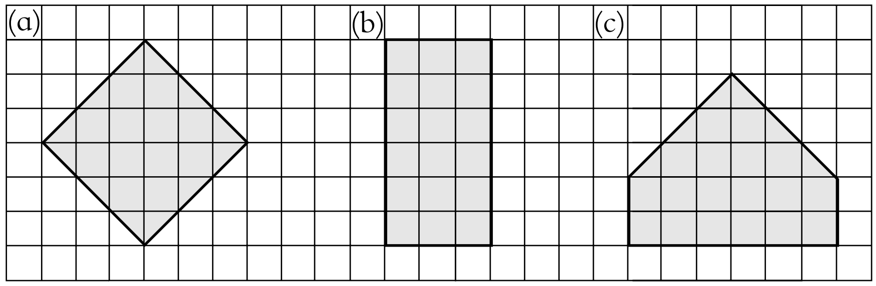
(d) On a copy of the grid opposite, shade in two more squares to make a shape which has dashed line as a line of symmetry. You may use a mirror or tracing paper to help you.



Rotating Shapes and Rotational Symmetry

Example 1

State the order of rotational symmetry of each of the following shapes:



Solution

(a) Order 4. This means that the shape can be rotated 4 times about its centre before returning to its starting position. Each rotation will be through an angle of 90°, and after each one the rotated shape will occupy the same position as the original square.

(b) Order 2.

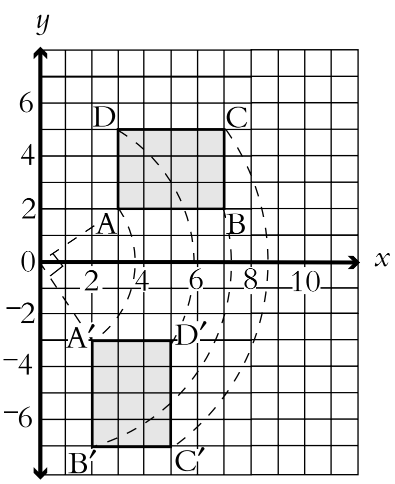
(c) Order 1. This means that the shape does not have rotational symmetry.

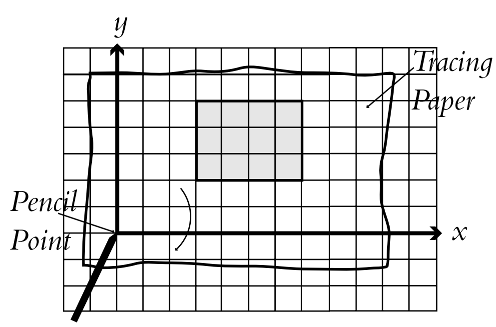
Example 2

The corners of a rectangle have co-ordinates (3, 2), (7, 2), (7, 5) and (3, 5). The rectangle is to be rotated through 90° clockwise about the origin. Draw the original rectangle and its position after being rotated.

Solution

The following diagram shows the original rectangle ABCD and the rotated rectangle A′B′C′D′. The curves show how each corner moves as it is rotated. An easy way to rotate a shape is to place a piece of tracing paper over the shape, trace the shape, and then rotate the tracing paper about the centre of rotation, as shown.





Tracing paper placed over shape, and then rotated around centre of rotation.

Example 3

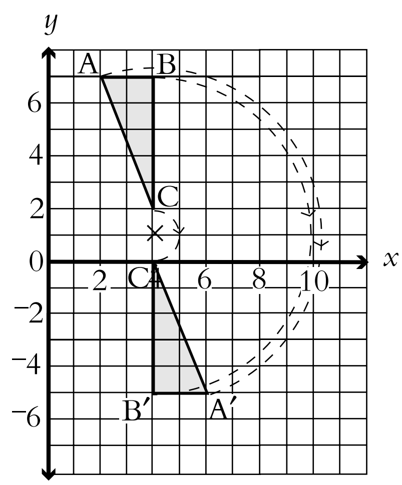
A triangle has corners at the points with co-ordinates (4, 7), (2, 7) and (4, 2).

(a) Draw the triangle.

(b) Rotate the triangle through 180° about the point (4, 1).

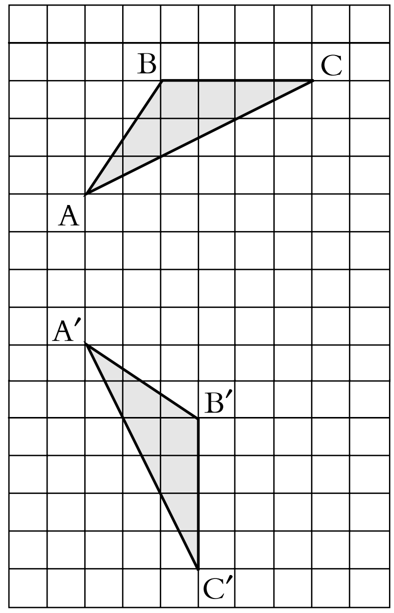
Solution

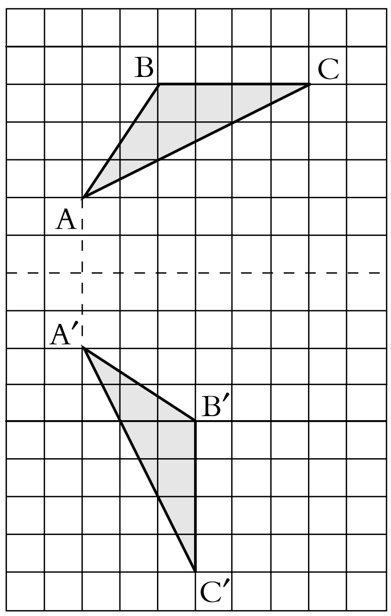
The diagram shows how the original triangle ABC is rotated about the point (4, 1) to give the triangle A′B′C′.



Example 4

The diagram shows the triangle ABC which is rotated through 90° to give A′B′C′.
Determine the position of the centre of the rotation.





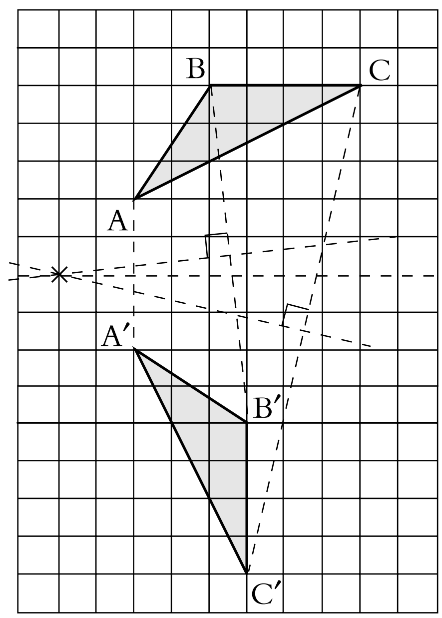
Solution

The first step is to join the points A and A′ and draw the perpendicular bisector of this line.

The centre of rotation must be on this line.

Repeat the process, drawing the perpendicular bisectors of BB′ and CC′ as shown opposite.

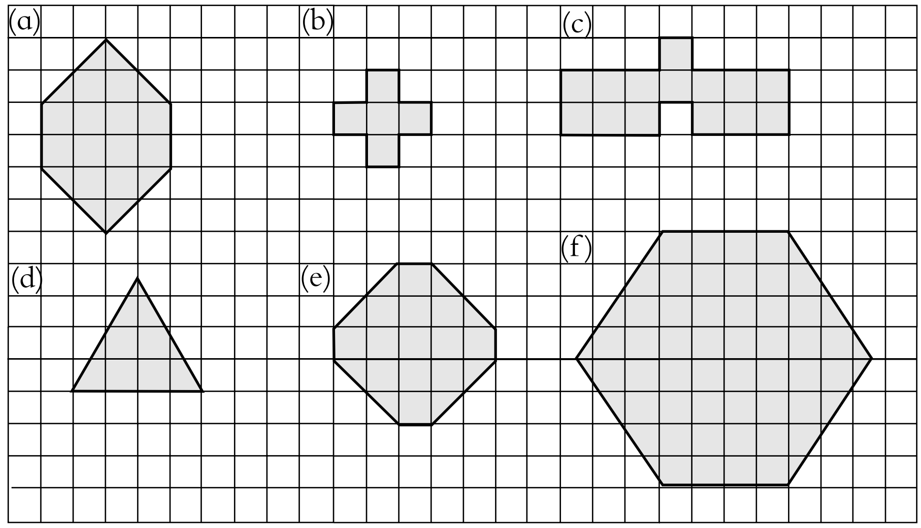
The point where the lines cross is the centre of rotation.



Note: For simple rotations you may be able to spot the centre of rotation without having to use the method shown above. Alternatively, you may be able to find the centre of rotation by experimenting with tracing paper.

Skill Exercises: Rotating Shapes and Rotational Symmetry

1. State the order of rotational symmetry of each of the following shapes:



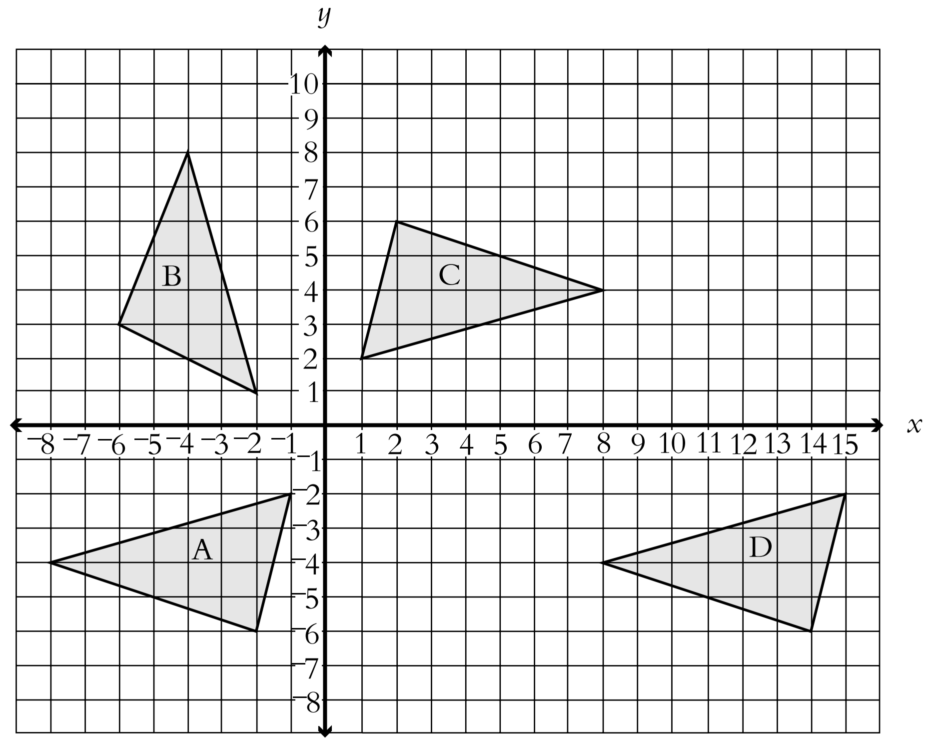
2. Which of the capital letters in the English alphabet have rotational symmetry?

3. A rectangle has corners at the points A(2, 4), B(6, 4), C(6, 6) and D(2, 6).
   (a) Draw this rectangle.
   (b) Rotate the rectangle through 90° clockwise about the point (0, 0).
   (c) Rotate the rectangle ABCD through 180° about the point (0, 0).

4. Rotate the rectangle formed by joining the points (1, 1), (3, 1), (3, 2) and (1, 2) through 90° clockwise about the origin.

5. A triangle has corners at the points with co-ordinates (4, 7), (3, 2) and (5, 1). Find the co-ordinates of the triangles that are obtained by rotating the original triangle:
   (a) through 90° anticlockwise about (0, 3)
   (b) through 180° about (4, 0)
   (c) through 90° clockwise about (6, 2)

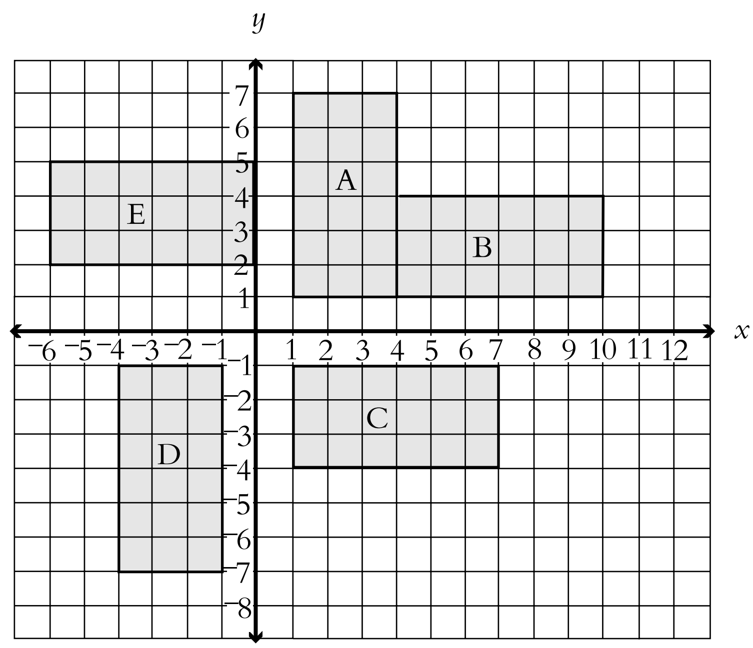
6. The following diagram shows the triangles A, B, C and D:



Describe the rotation that takes:

(a) A to B (b) A to C (c) C to B (d) C to D

7. The following diagram shows the rectangles A, B, C and D:



Describe the rotation that takes:

(a) A to B (b) A to C (c) A to D (d) A to E

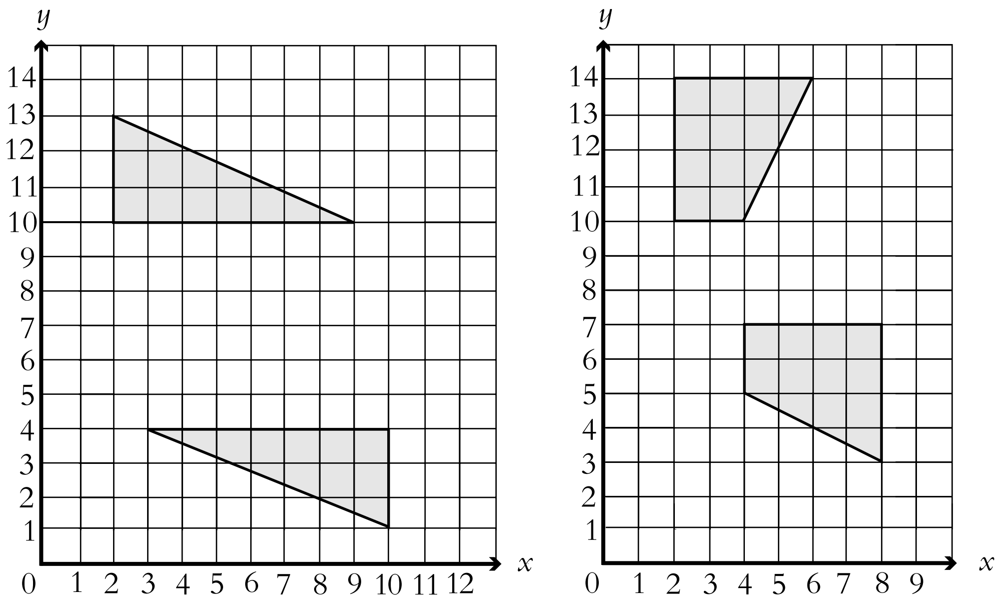
8. The triangle A has corners at the points with co-ordinates (1, 7), (3, 6) and (2, 4).

   (a) Rotate triangle A through 180° about the origin to get triangle B.

   (b) Rotate triangle B clockwise through 90° about the point $(0, ^{-}4)$ to get triangle C.

   (c) Write down the co-ordinates of the corners of triangle C.

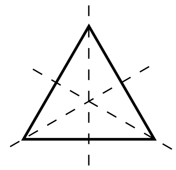
9. The following diagrams show two rotations. Find the co-ordinates of the centre of rotation in each case.



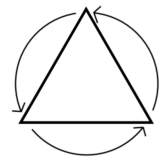
10. A triangle has corners at the points A(4, 2), B(6, 3) and C(5, 7). The triangle is rotated to give the triangle with corners at the points $A'(3, ^{-}1)$, $B'(4, ^{-}3)$ and $C'(8, ^{-}2)$.

    Describe fully this rotation.

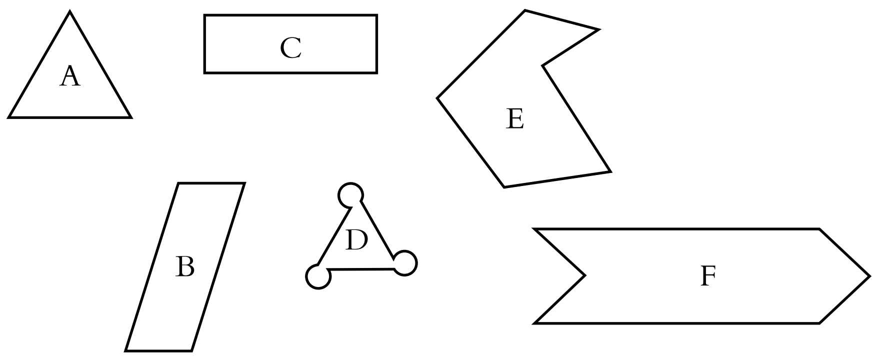
11. An equilateral triangle has three lines of symmetry.



It has rotational symmetry of order 3.



Copy and complete the table below. You may use a mirror or tracing paper to help. The letters for the first two shapes have been written for you.

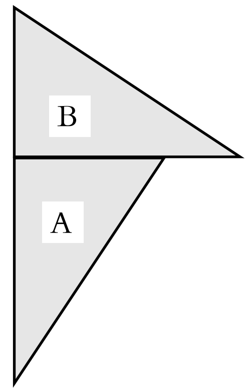


Number of Lines of Symmetry

		0	1	2	3	
Order of Rotational Symmetry	1					
	2	B				
	3				A	

12. (a) You can rotate triangle A onto triangle B.

    Make a copy of the diagram and put a cross on the centre of rotation. You may use tracing paper to help you.



    (b) You can rotate triangle A onto triangle B.

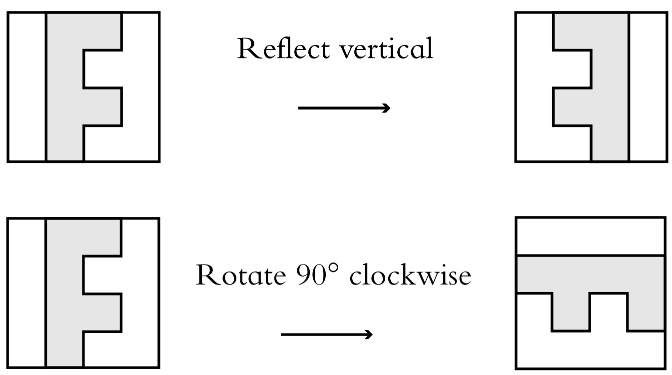
    The rotation is anti-clockwise. What is the angle of rotation?

13. Julie has written a computer programme to transform pictures of tiles. There are only two instructions in her programme,

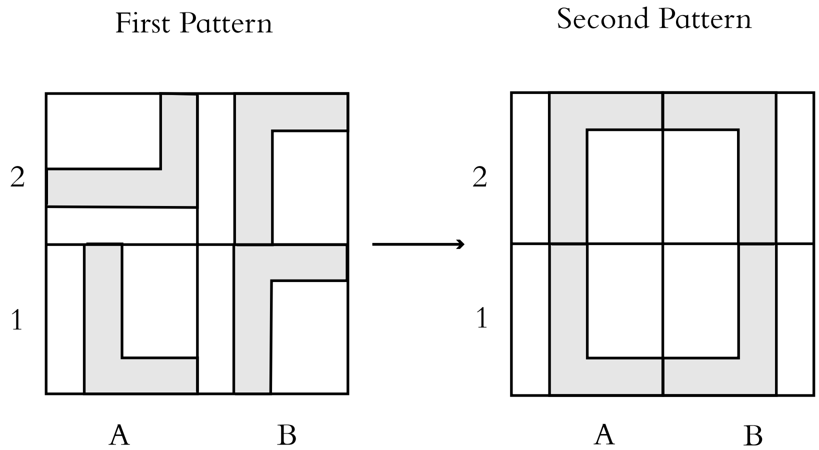
reflect vertical

or

rotate 90° clockwise



(a) Julie want to transform the first pattern to the second pattern:



Copy and complete the following instructions to transform the tiles B1 and B2. You must use only reflect vertical or rotate 90° clockwise.

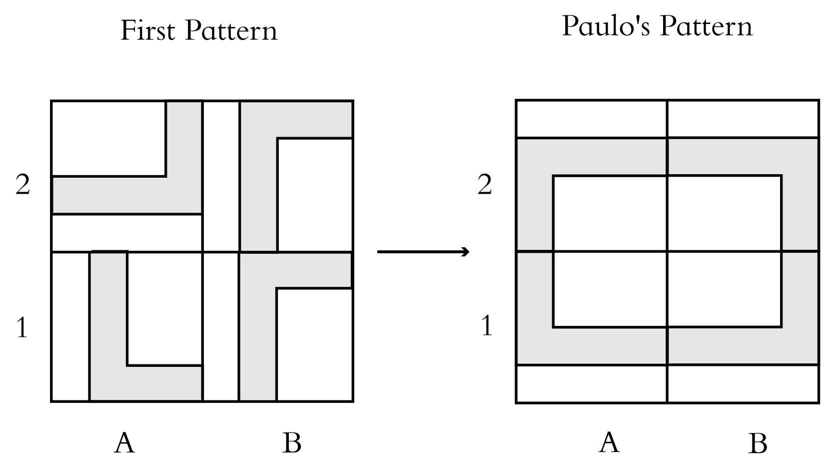
A1 Tile is in the correct position.

A2 Reflect vertical, and then rotate 90° clockwise.

B1 Rotate 90° clockwise and then ....

B2 ....

(b) Paulo starts with the first pattern that was on the screen:



Copy and complete the instructions for the transformations of A2, B1 and B2 to make Paulo's pattern. You must use only reflect vertical or rotate 90° clockwise.

A1 Reflect vertical, and then rotate 90° clockwise.

A2 Rotate 90° clockwise, and then ....

B1 ....

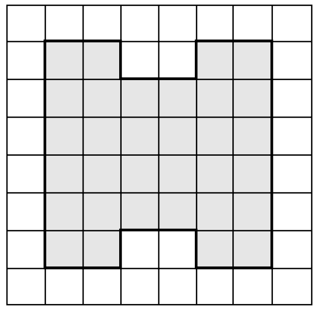
B2 ....

Examining the Symmetry of Polygons

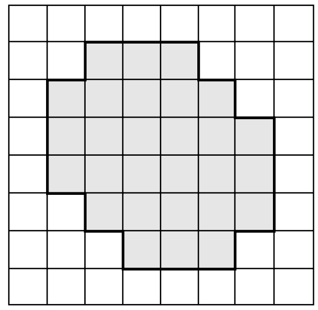
Example 1

Draw the lines of symmetry of each shape below:

(a)

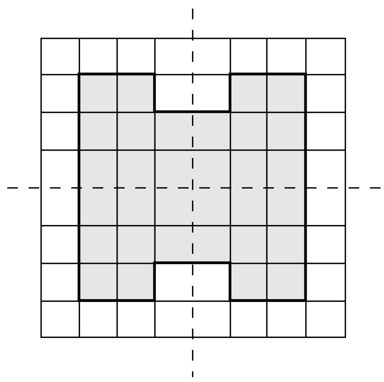


(b)

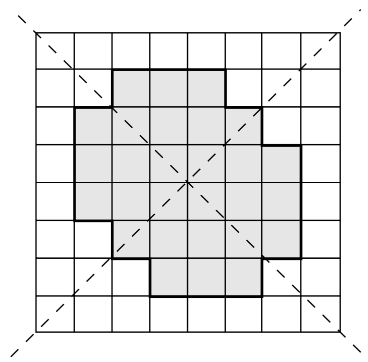


Solution

(a) The shape has two lines of symmetry, one horizontal and the other vertical, as shown below:



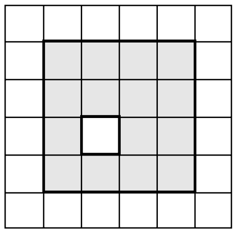
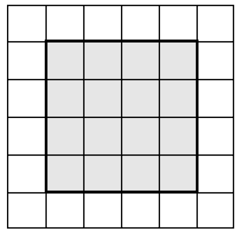
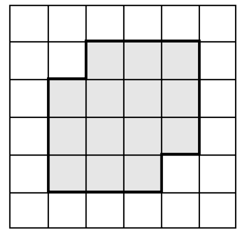
(b) The shape has two diagonal lines of symmetry, as shown below:



Reminder: The order of rotational symmetry is the number of times in one rotation of 360° that a shape is identical to that of its starting position.

Example 2

What is the order of rotational symmetry of each of the following shapes:

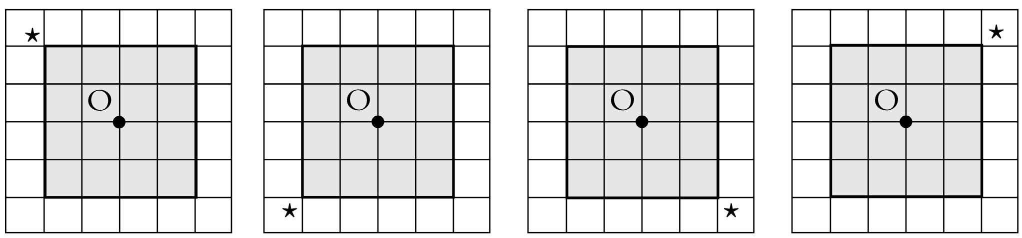
(a)  (b)  (c) 

Solution

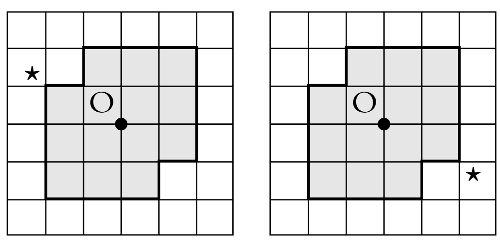
(a) The shape has rotational symmetry of order 1, meaning that it does not have rotational symmetry. (The shape cannot be rotated to another position within 360° and still look the same.)

(b) The shape has rotational symmetry of order 4.

The following diagram shows how the position of one corner, marked ⋆, moves as the square is rotated anticlockwise about its centre.



(c) The shape has rotational symmetry of order 2. The diagram shows the position of a corner, marked ⋆, as the shape is rotated about its centre O.



Example 3

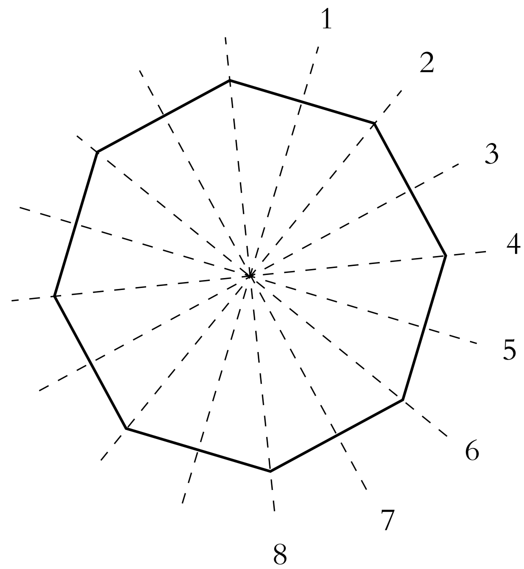
An octagon is a shape which has eight sides.

(a) Draw a diagram to show the lines of symmetry of a regular octagon.

(b) What is the order of rotational symmetry of a regular octagon?

Solution

(a) A regular octagon has eight lines of symmetry, as shown in the following diagram:



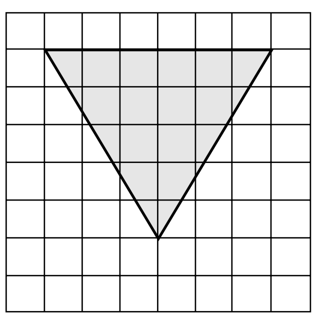
(b) A regular octagon has rotational symmetry of order 8.

The order of rotational symmetry and the number of lines of symmetry of any regular polygon is equal to the number of sides.

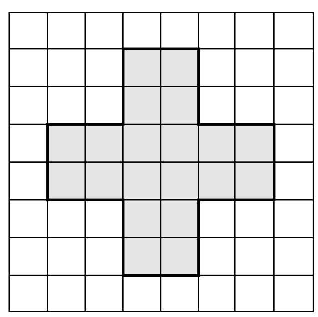
Skill Exercises: Examining the Symmetry of Polygons

1. Copy each of the following shapes and draw in all the lines of symmetry. For each one, state the order of rotational symmetry and mark on your copy its centre of rotation, O.

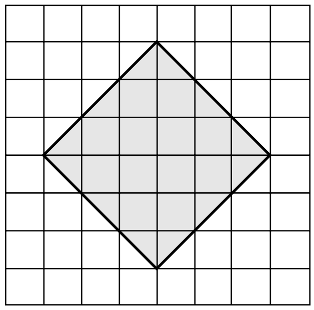
(a)



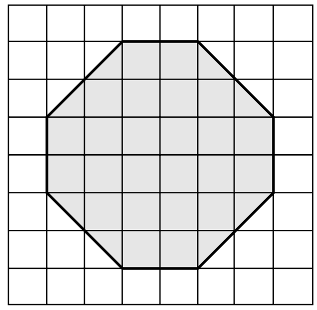
(b)



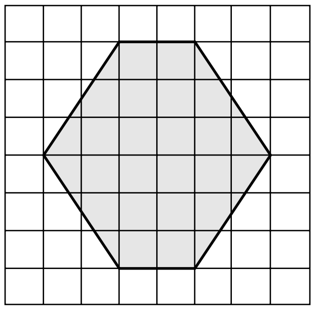
(c)



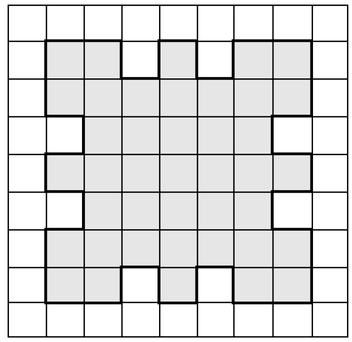
(d)



(d)

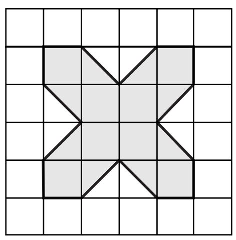


(f)

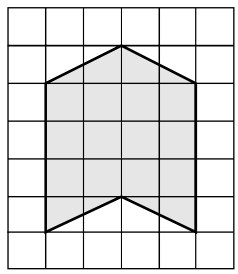


2. State the order of rotational symmetry and the number of lines of symmetry, for each of the following shapes:

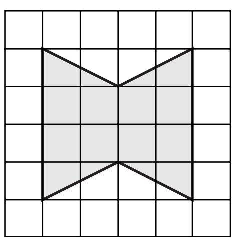
(a)



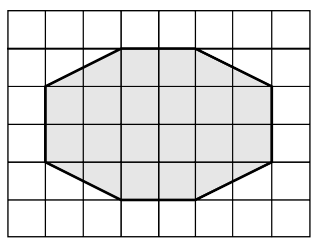
(b)



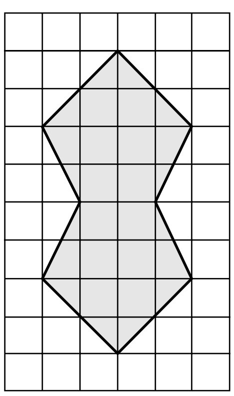
(c)



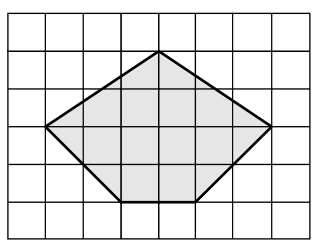
(d)



(e)

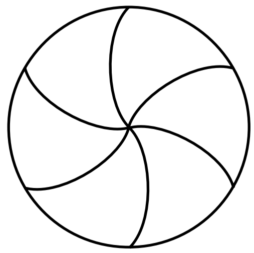


(f)

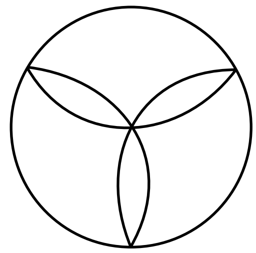


3. Describe fully the symmetries of the following shapes:

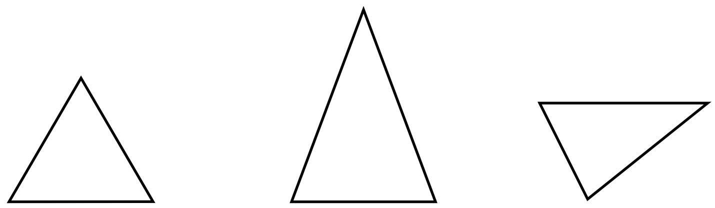
(a)



(b)



4. Describe the symmetry properties of each of the following triangles:



Equilateral Isosceles Scalene

5. (a) How many lines of symmetry does a square have? Draw a diagram to show this information.

   (b) What is the order of rotational symmetry of a square?

6. (a) Copy and complete the following table:

Shape	Order of Rotational Symmetry	Number of Lines of Symmetry
Equilateral triangle		
Square		
Regular pentagon		
Regular hexagon		
Regular heptagon (7 sides)		
Regular octagon		
Regular nonagon (9 sides)		
Regular decagon (10 sides)		
Regular dodecagon (12 sides)		

   (b) What do you conclude from the information in the table?

7. Draw a shape that has no lines of symmetry, but has rotational symmetry of order 3?

8. Draw a shape that has at least one line of symmetry and no rotational symmetry.

9. Draw two regular polygons, one with an even number of sides and one with an odd number of sides. By drawing lines of symmetry on each diagram, show how the positions of the lines of symmetry differ between odd- and even-sided regular polygons.

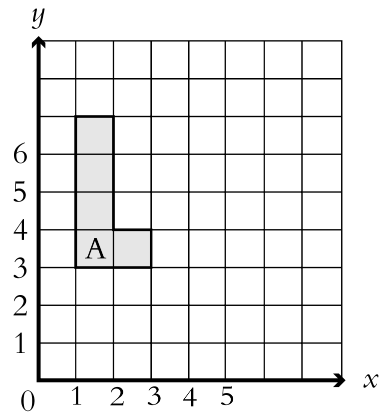
10. Draw an irregular polygon that has both line and rotational symmetry. Show the lines of symmetry and the centre of rotation, O, and state its order of rotational symmetry.

Combining Transformations

Example 1

The shape shown in the diagram is reflected first in the y-axis and its image is then reflected in the x-axis.

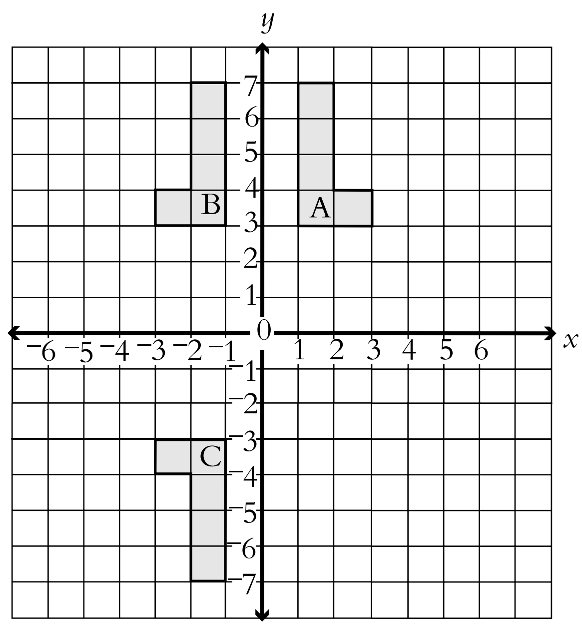
What single transformation would have the same result as these two transformations?



Solution

The diagram shows how the original shape A is first reflected to B, and B is then reflected to C.

A rotation of 180° about the origin would take A straight to C.

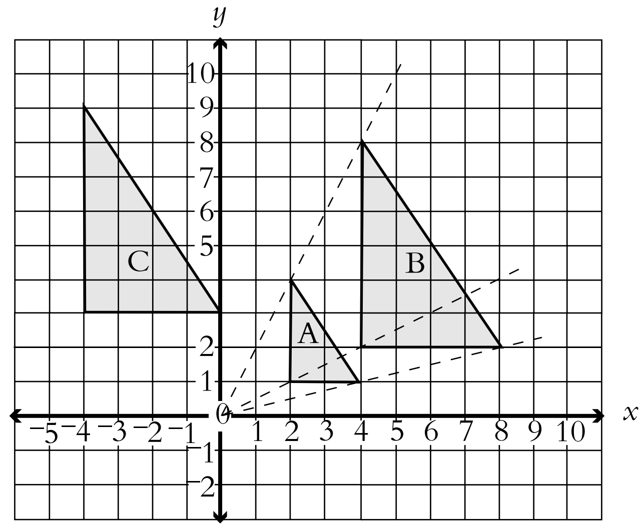


Example 2

A triangle is to be enlarged with scale factor 2, using the origin as the centre of enlargement. Its image is then to be translated along the vector $\begin{bmatrix} ^-8 \\ 1 \end{bmatrix}$.

The co-ordinates of the corners of the triangles are (2, 1), (2, 4) and (4, 1). What single transformation would have the same result?

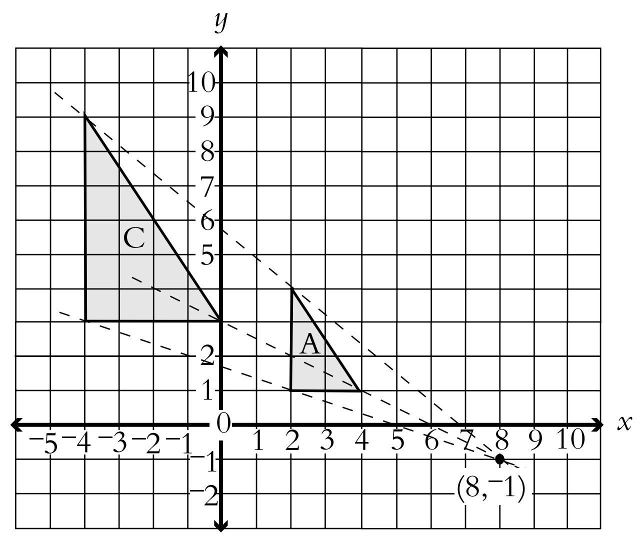
Solution



The diagram shows the original triangle A; the enlargement takes it to B, which is then translated to C.

The triangle A could be enlarged with scale factor 2 to give C.

This diagram shows that the centre of enlargement would be the point $(8, {}^-1)$:

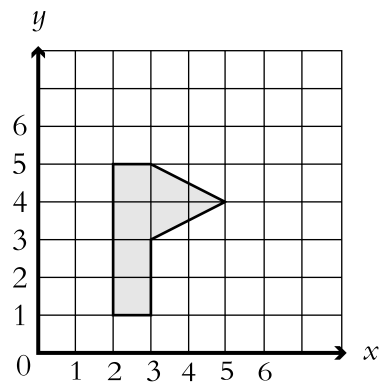


The single transformation that will move triangle A to triangle C is an enlargement, scale factor 2, centre $(8, {}^-1)$.

Skill Exercises: Combining Transformations

1. (a) Reflect the shape shown in the x-axis and then reflect its image in the y-axis.

   (b) What single transformation would have the same result as these two transformations?



2. A rectangle has corners at the points with co-ordinates $(1, 2)$, $(3, 1)$, $(5, 5)$ and $(3, 6)$. It is first reflected in the x-axis and then its image is rotated through 180° about the origin.

   Describe how to move the rectangle from its original position to its final position, using only one transformation.

3. A shape is rotated through 180° about the origin and then its image is reflected in the x-axis.

   (a) Choose a shape and carry out the transformations described above.

   (b) What single transformation would have the same result as the two transformations described above?

4. A triangle has corners at the points with co-ordinates $(2, 2)$, $(3, 6)$ and $(8, 6)$.

   (a) Draw the triangle and enlarge it with scale factor 2, using the origin as the centre of enlargement.

   (b) Translate the enlarged shape along the vector $\begin{bmatrix} -3 \\ -6 \end{bmatrix}$.

   (c) Describe fully the enlargement that would produce the final triangle from the original triangle.

5. (a) Draw the triangle with corners at the points $(2, 1)$, $(4, 1)$ and $(4, 2)$.

   (b) Reflect this shape in the line $y = x$.

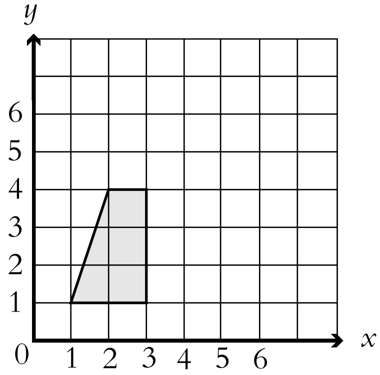
   (c) Reflect the new triangle in the y-axis.

   (d) What single transformation would have the same result as the two transformations described above?

6. (a) Reflect a shape of your choice in the line $y = x$ and then reflect the image in the line $y = {}^{-}x$.

   (b) Describe a single transformation that would have the same result.

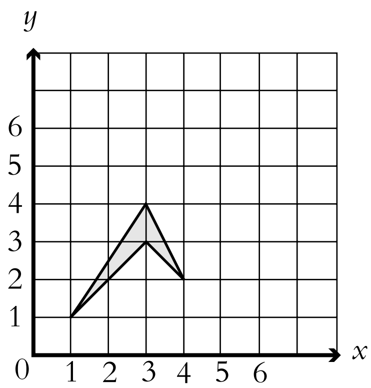
7. The shape shown in the following diagram is to be reflected in the line $x = 4$ and then its image is to be reflected in the line $y = 5$.



   (a) Draw a diagram to show how the shape moves.

   (b) What single transformation would have the same result?

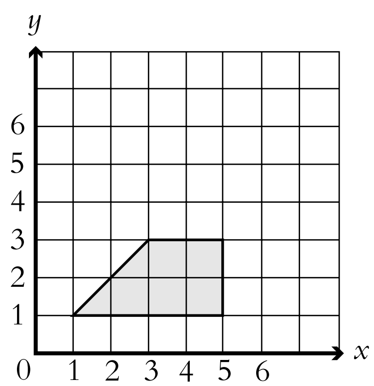
8. The shape shown in the diagram is translated along the vector $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$.



   (a) Draw the final position of the shape.

   (b) Describe how the shape could be moved to this position using two reflections.

9. The shape shown in the diagram is to be enlarged with scale factor 3 using the point (0, 4) as the centre of enlargement.



   (a) Draw the enlarged shape.

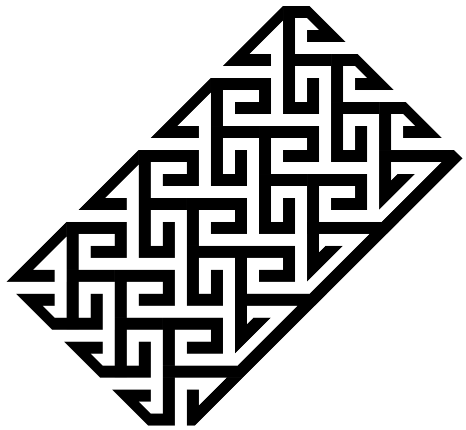
(b) The enlarged shape is translated along the vector $\begin{bmatrix} 0 \\ 8 \end{bmatrix}$. Draw the new position of the shape.

(c) Describe the single enlargement that would have the same result as the two transformations used opposite.

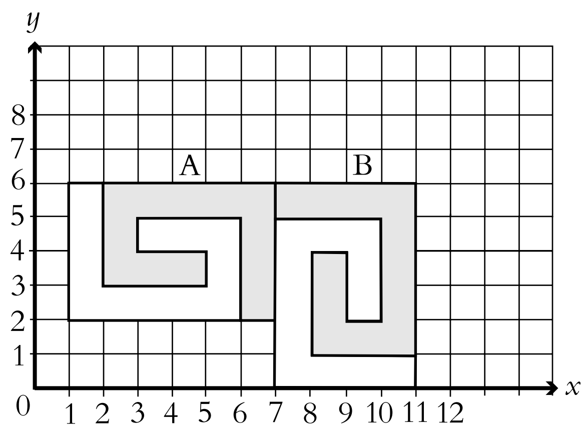
10. A shape is reflected in the line $y = x$, then in the line $y = {}^{-}x$, and finally in the x-axis.

What single transformation would have the same result?

11. The following design is based on a traditional pattern.



Part of the pattern is shown below:



The pattern is made of two rectangular blocks, A and B.

Use two transformations to map block A onto block B. Your transformations must be either rotations or reflections.

Mark any mirror lines or centres of rotation on a copy of the previous diagram.

Write down instructions for the first and second transformations.

Give co-ordinates of any centres of rotation, the amount of turn and direction of turn. Give the equations of any lines of reflection.

Unit 6: ANSWERS – TRIGONOMETRY

Section 6.1 Using Pythagoras' Theorem With Right-Angled Triangles

(Pg. 9) Skill Exercises: Using Pythagoras' Theorem

1. (a) 10 cm (b) 20.40 cm (c) 12.04 cm (d) 20.25 cm

2. (a) 4.47 cm (b) 8.72 m (c) 7.94 m (d) 9.64 m

3. (a) AB = 9 cm (b) EF = 9.1 m
   (c) GH = 100 km (d) JK = 36 m

4. (a) and (c) are the only right-angled triangles

5. 141 m (to the nearest m) or 140 m (to the nearest 10 m)

6. 25.8 cm (to the nearest mm)

7. (a) 2.01 m (to the nearest cm) (b) 2.02 m (to the nearest cm)

8. Perimeter = 44 cm Area = 84 cm^2

9. 8.66 m (to the nearest cm) or 8.70 m (to the nearest 10 cm)

10. 4.58 m (to the nearest cm) or 4.60 m (to the nearest 10 cm)

11. AC $= \sqrt{700} + \sqrt{5500}$
    $= 26.45751311 + 74.16198487$
    $= 100.619498$ cm
    $= 101$ cm (to the nearest cm)

12. $$\begin{aligned}(\text{distance})^2 &= 7.5^2 + 4.8^2\\ AC^2 &= 7.5^2 + 4.8^2\\ &= 56.25 + 23.04\\ &= 79.29\\ \therefore \text{distance} &= \sqrt{79.29}\\ &= 8.904493248 \text{ km}\\ &= 8.90 \text{ km (to 3 significant figures)}\end{aligned}$$

13. (a) $$\begin{aligned}(\text{length of strut})^2 &= 150^2 + 190^2\\ &= 22\,500 + 36\,100\\ &= 58\,600\\ \therefore \text{length of strut} &= \sqrt{58\,600}\\ &= 242.0743687 \text{ cm}\\ &= 242 \text{ cm (to the nearest cm)}\end{aligned}$$

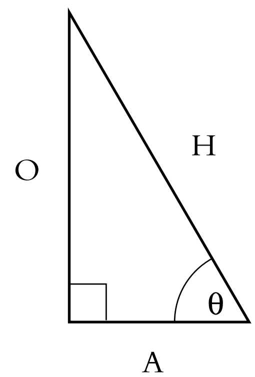
(b) $$\begin{aligned}(\text{diagonal of rectangle})^2 &= 150^2 + 80^2\\ &= 22\,500 + 6400\\ &= 28\,900\\ \therefore \text{diagonal of rectangle} &= \sqrt{28\,900}\\ &= 170 \text{ cm}\end{aligned}$$

Since 170 cm > 165 cm, i.e. diagonal of rectangle > width of room, the room is not wide enough for the cabinet to be moved to its new position.

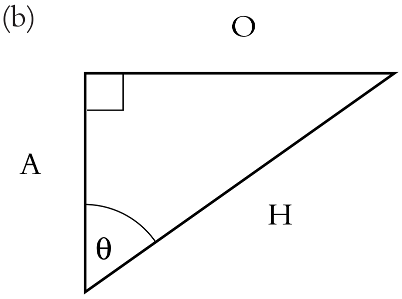
Section 6.2 Using Trigonometric Ratios With Right-Angled Triangles

(Pg. 16) Skill Exercises: Defining Trigonometric Ratios

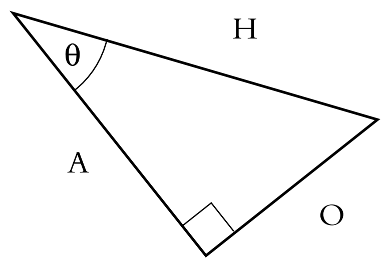
1. (a)



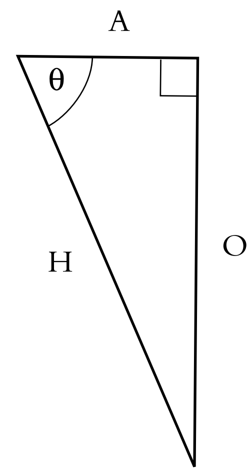
(b)



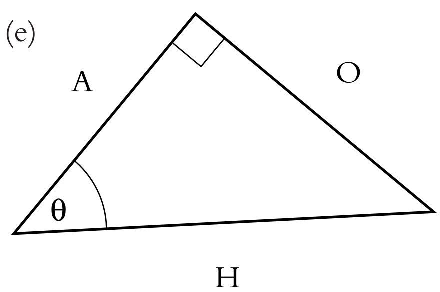
(c)



(d)



(e)



2. (a) $\sin\theta° = \frac{8}{10}$ $\cos\theta° = \frac{6}{10}$ $\tan\theta° = \frac{8}{6}$

(b) $\sin\theta° = \frac{20}{29}$ $\cos\theta° = \frac{21}{29}$ $\tan\theta° = \frac{20}{21}$

(c) $\sin\theta° = \frac{5}{13}$ $\cos\theta° = \frac{12}{13}$ $\tan\theta° = \frac{5}{12}$

(d) $\sin\theta° = \frac{8}{17}$ $\cos\theta° = \frac{15}{17}$ $\tan\theta° = \frac{8}{15}$

(e) $\sin\theta° = \frac{12}{15}$ $\cos\theta° = \frac{9}{15}$ $\tan\theta° = \frac{12}{9}$

(Pg. 19) Skill Exercises: Estimating Trigonometric Ratios

1. (a) Own constructions
   (b) Answers should be approximately 0.87
   (c) Answers should be approximately 0.5
   (d) Answers should be approximately 1.73

2. (a) Own constructions
   (b) Answers should be approximately:
   (i) 0.64 (ii) 0.77 (iii) 1.19

3. (a) Own constructions
   (b) sin 45° = cos 45° because the triangle is isosceles and the opposite and adjacent sides have the same length. This means that tan 45° = 1.

4. Own constructions

   Their values, rounded to 2 significant figures, should be close to the answer for part (b) as given below:

(b)

Angle	sine	cosine	tangent
10°	0.17	0.98	0.18
20°	0.34	0.94	0.36
30°	0.50	0.87	0.58
40°	0.64	0.77	0.84
50°	0.77	0.64	1.19
60°	0.87	0.50	1.73
70°	0.94	0.34	2.75
80°	0.98	0.17	5.67

5. 30°

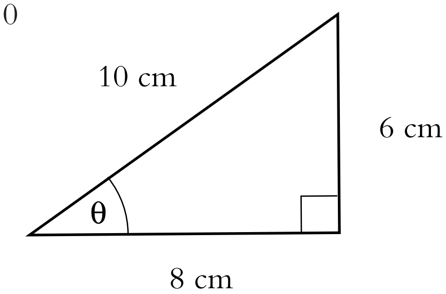
6. 80.21218094° = 80° (to the nearest degree)

7. (a) $\cos \theta° = 1$ (b) $\sin \theta° = 0$ (c) $\sin 90° = 1$
   (d) $\cos 90° = 0$ (e) $\tan 0° = 0$ (f) undefined

8. (a) sin 82° = 0.990 (b) cos 11° = 0.982 (c) sin 42° = 0.669
   (d) tan 80° = 5.67 (e) tan 52° = 1.28 (f) tan 38° = 0.781

9. (a) 72.5° (b) 50.4° (c) 58.3°
   (d) 18.1° (e) 27.1° (f) 85.0°

10. (a) Two triangles are 3, 4, 5 or 6, 8, 10 right-angled triangles. There are also many enlargements of these.



From the triangle, $\sin \theta = 0.6$

$\tan \theta = 0.75$

(b) $\theta = \cos^{-1}(0.8)$

$= 36.86989765°$

$= 36.9°$ (to 1 decimal place)

(c) $\sin 36.86989765° = 0.6$

$\tan 36.86989765° = 0.75$

(Pg. 23) Skill Exercises: Calculating the Length of a Side

1. (a) 8.4 cm (b) 12.7 cm (c) 6.3 cm (d) 13.5 cm
2. (a) 14.2 cm (b) 12.1 cm (c) 2.2 cm (d) 29.0 cm
3. (a) $x = 12 \times \cos 50°$ $= 7.713451316$ cm $= 7.71$ cm (to 3 s.f.)

   $y = 12 \times \sin 50°$ $= 9.192533317$ cm $= 9.19$ cm (to 3 s.f.)

   (b) $a = 40 \times \cos 35°$ $= 32.76608177$ m $= 32.8$ m (to 3 s.f.)

   $b = 40 \times \sin 35°$ $= 22.94305745$ m $= 22.9$ m (to 3 s.f.)

   (c) $x = 8 \times \tan 40°$ $= 6.712797049$ cm $= 6.71$ cm (to 3 s.f.)

   (d) $p = 11 \times \tan 20°$ $= 4.003672577$ cm $= 4.00$ cm (to 3 s.f.)

   (e) $z = 9 \times \cos 25°$ $= 8.156770083$ cm $= 8.16$ cm (to 3 s.f.)

   (f) $x = 20 \times \cos 70°$ $= 6.840402866$ m $= 6.84$ m (to 3 s.f.)

4. (a) hypotenuse $= 8 \div \sin 35°$
$= 13.94757436$ cm
$= 13.9$ cm (to 3 s.f.)

(b) hypotenuse $= 3 \div \cos 70°$
$= 8.771413201$ cm
$= 8.77$ cm (to 3 s.f.)

(c) hypotenuse $= 18 \div \sin 41°$
$= 27.43655556$ cm
$= 27.4$ cm (to 3 s.f.)

(d) hypotenuse $= 33 \div \cos 42°$
$= 44.40588008$ cm
$= 44.4$ cm (to 3 s.f.)

5. (a) $a = 17 \times \sin 50°$
$= 13.02275553$ cm
$= 13.02$ cm (to 2 d.p.)

$b = 17 \times \cos 50°$
$= 10.92738936$ cm
$= 10.93$ cm (to 2 d.p.)

(b) $c = 15 \div \tan 25°$
$= 32.16760381$ cm
$= 32.17$ cm (to 2 d.p.)

$d = 15 \div \sin 25°$
$= 35.49302375$ cm
$= 35.49$ cm (to 2 d.p.)

(c) $e = 14 \times \tan 12°$
$= 2.975791863$ cm
$= 2.98$ cm (to 2 d.p.)

$f = 14 \div \cos 12°$
$= 14.31276833$ cm
$= 14.31$ cm (to 2 d.p.)

(d) $g = 6 \times \tan 62°$
$= 11.28435879$ cm
$= 11.28$ cm (to 2 d.p.)

$h = 6 \div \cos 62°$
$= 12.78032681$ cm
$= 12.78$ cm (to 2 d.p.)

6. (a) Distance from foot of the wall $= 6 \times \cos 65°$
$= 2.53570957$ m
$= 2.54$ m (to the nearest cm)

(b) Height of top above the ground $= 6 \times \sin 65°$
$= 5.437846722$ m
$= 5.44$ m (to the nearest cm)

7. (a) Distance east $= 50 \times \sin 70°$
$= 46.98463104$ km
$= 47$ km (to the nearest km)
(b) Distance north $= 50 \times \cos 70°$
$= 17.10100717$ km
$= 17$ km (to the nearest km)

8. Perimeter $= 15 + (15 \times \sin 20°) + (15 \times \cos 20°)$
$= 34.22569146$ cm
$= 34.23$ cm (to 2 d.p.)
Area $= (15 \times \sin 20°) \times (15 \times \cos 20°) \div 2$
$= 36.15680304\ \text{cm}^2$
$= 36.16\ \text{cm}^2$ (to 2 d.p.)

9. Height of top of ramp $= 6 \times \sin 50° = 4.596266659$ m
$= 4.60$ m (to the nearest cm)

10. (a) Length of rope $= 6 \div \sin 19° = 18.42932092$ m
$= 18.43$ m (to the nearest cm)
(b) Height of window $= 6 \div \tan 19° = 17.42526527$ m
$= 17.43$ m (to the nearest cm)

(Pg. 28) Skill Exercises: Calculating the Size of an Angle

1. (a) 53.1° (b) 60°
(c) 45° (d) 17.5°
(e) 90° (f) 85.9°

2. (a) 72.6° (b) 30.4°
(c) 35.5° (d) 43.5°

3. (a) $\theta = \sin^{-1}(0.7)$
$= 44.427004°$
$= 44.4°$ (to 1 d.p.)
(b) $\theta = \cos^{-1}(0.8)$
$= 36.86989765°$
$= 36.9°$ (to 1 d.p.)
(c) $\theta = \cos^{-1}(0.954545454)$
$= 17.3414428°$
$= 17.3°$ (to 1.d.p.)

(d) $\theta = \sin^{-1}(0.588235294)$

$= 36.03187907°$

$= 36.0°$ (to 1 d.p.)

4. (a) $\theta = \sin^{-1}(0.7)$

$= 44.427004°$ (to 1 d.p.)

$= 44.4°$

(b) $\theta = \tan^{-1}(0.875)$

$= 41.18592517°$

$= 41.2°$ (to 1 d.p.)

(c) $\theta = \cos^{-1}(0.666\ 666\ 666)$

$= 48.1896851°$

$= 48.2°$ (to 1 d.p.)

(d) $\theta = \tan^{-1}(3)$

$= 71.56505118°$

$= 71.6°$ (to 1 d.p.)

5. 37°, 53° and 90°

6. Angle of slope $= \tan^{-1}(0.147058823)$

$= 8.365886124°$

$= 8°$ (to the nearest degree)

7. Angle with ground $= \cos^{-1}(0.5)$

$= 60°$

8. (a) (Length of diagonal)$^2 = 12^2 + 18^2$

$= 144 + 324$

$= 468$

Length of diagonal $= \sqrt{468}$

$= 21.63330765$ cm

$= 21.6$ cm (to the nearest mm)

(b) Angle between diagonal and shorter side $= \tan^{-1}\left(\frac{12}{18}\right)$

$= \tan^{-1}(1.5)$

$= 56.30993247°$

$= 56°$ (to the nearest degree)

9. Angle between flight path and horizontal = $\sin^{-1}\left(\frac{500}{3000}\right)$

$= \sin^{-1}(0.1666666)$

$= 9.594068227°$

$= 10°$ (to the nearest degree)

10. Angle between strings $= a° + b°$

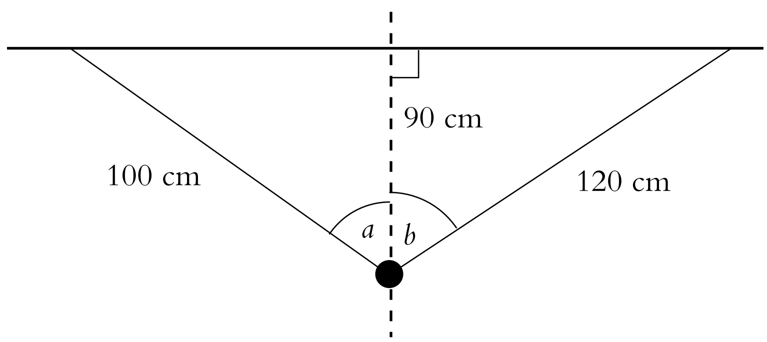
$= \cos^{-1}\left(\frac{90}{100}\right) + \cos^{-1}\left(\frac{90}{120}\right)$

$= \cos^{-1}(0.9) + \cos^{-1}(0.75)$

$= 25.84193276° + 41.40962211°$

$= 67.25155487°$

$= 67°$ (to the nearest degree)



(Pg. 32) Skill Exercises: Solving Problems using Trigonometric Ratios

1. (a) Using Pythagoras

$$a^2 = b^2 + c^2$$
$$10^2 = 9.85^2 + h^2$$
$$100 = 97.0225 + h^2$$
$$100 - 97.0225 = h^2$$
$$2.9775 = h^2$$
$$h = \sqrt{2.9775}$$
$$h = 1.725543 \text{ m}$$
$$h = 1.73 \text{ m (2 d.p.)}$$

∴ The ramp is too high

(b)
$$C = \frac{A}{H}$$
$$\cos 3.5° = \frac{A}{10}$$
$$10 \times \cos 3.5° = A$$
$$A = 9.981347984$$
$$A = 9.98 \text{ m (2 d.p.)}$$

The base of the ramp is 9.98 m

(c)
$$T = \frac{O}{A}$$
$$\tan\theta = \frac{1}{20}$$
$$\tan\theta = 0.05$$
$$\theta = \tan^{-1}(0.05)$$
$$\theta = 2.862405°$$
$$\theta = 2.86° \text{ (2 d.p.)}$$

An angle of 2.86° is recommended

2. (a)
$$(\text{Shortest distance})^2 = 6^2 + 4^2$$
$$= 36 + 16$$
$$= 52$$
$$\text{Shortest distance} = \sqrt{52}$$
$$= 7.211102551 \text{ km}$$
$$= 7.2 \text{ km (to 1 d.p.)}$$

(b) Angle between direction and north
$$\tan\theta = \frac{6}{4}$$
$$= \tan^{-1}(1.5)$$
$$= 56.30993247°$$
$$\therefore \text{bearing} = 056° \text{ (to the nearest degree)}$$

(c)
$$(\text{Distance west})^2 = 2.5^2 - 1.2^2$$
$$= 6.25 - 1.44$$
$$= 4.81$$
$$\text{Distance west} = \sqrt{4.81}$$
$$= 2.19317122 \text{ km}$$
$$= 2.2 \text{ km (to 1 d.p.)}$$

3. (a) Angle between direction and north

$$\tan\theta = \frac{6}{5}$$

$$\theta = \tan^{-1}(1.2)$$

$$= 50.19442891^\circ$$

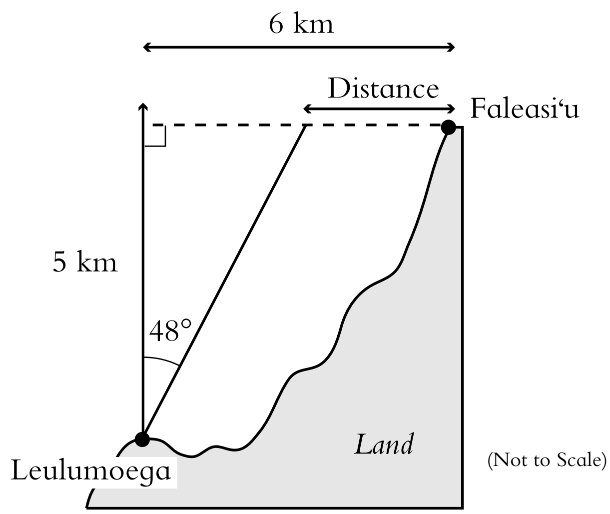
$$\therefore \text{bearing} = 050^\circ \text{ (to the nearest degree)}$$

(b) Distance north of Leulumoega $= \tan 48^\circ \times 5$

$= 5.55$ km (to 3 s.f.)

$\therefore$ distance west of Faleasi'u $= 6 \text{ km} - 5.55 \text{ km}$

$= 0.45$ km



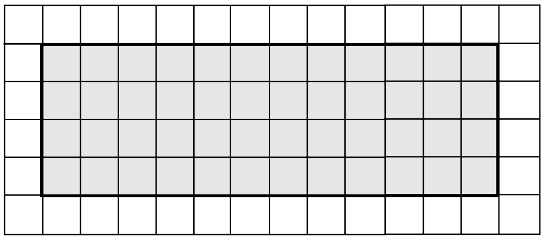
Unit 7: ANSWERS – GEOMETRY

Section 7.1 Exploring Transformations Using Enlargement

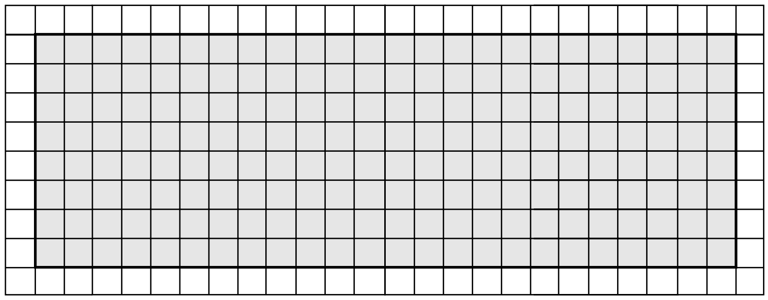
(Pg. 38) Skill Exercises: Drawing an Enlargement

1. B Scale factor 2
   E Scale factor 3
2. B, E
3. B Scale factor 2
   C Scale factor 3
   D Scale factor $\frac{1}{2}$ or 0.5
   E Scale factor $1\frac{1}{2}$ or 1.5
4. C, E
5. B, E
6. C, E

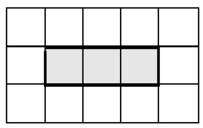
7. (a) Scale factor 2



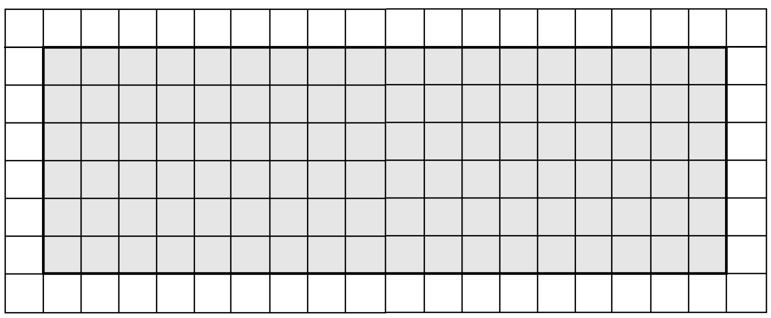
(b) Scale factor 4



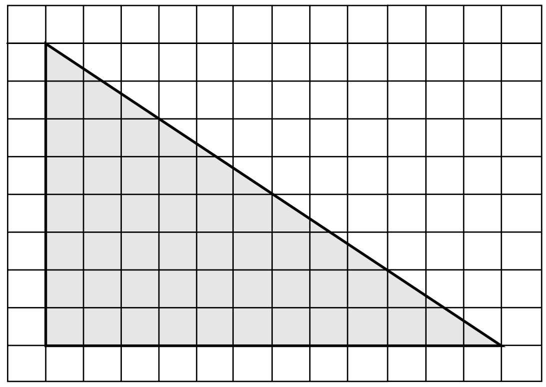
(c) Scale factor $\frac{1}{2}$



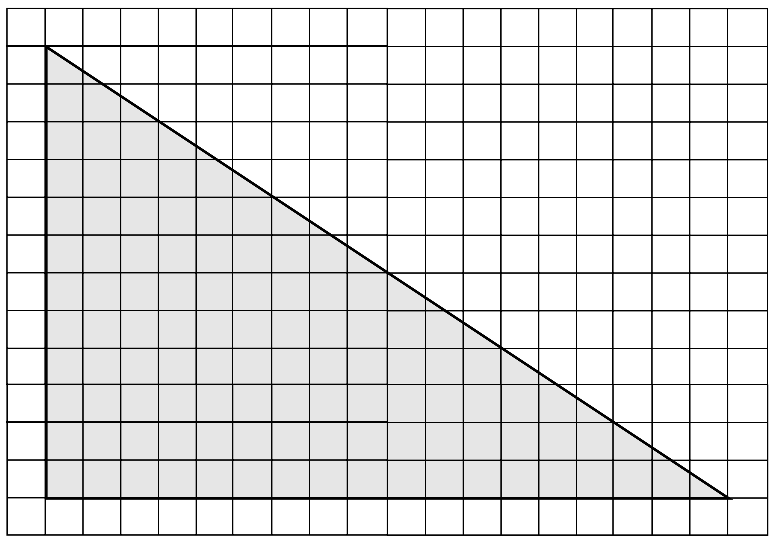
(d) Scale factor 3



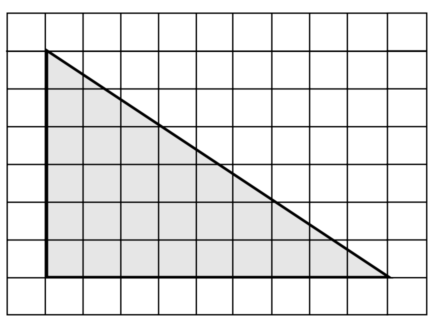
8. (a) Scale factor 2



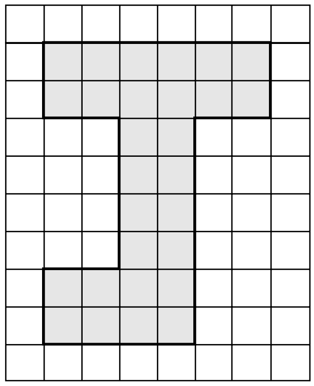
(b) Scale factor 3



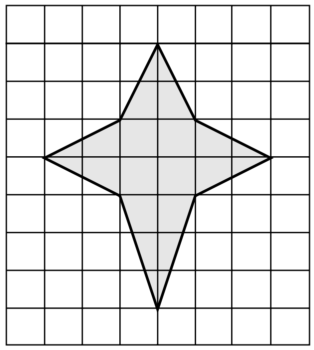
(c) Scale factor $1\frac{1}{2}$



9.

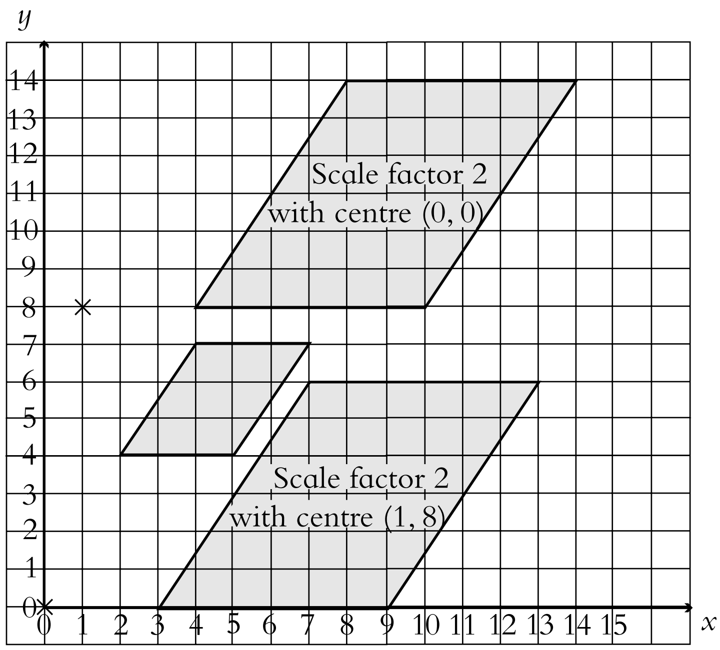


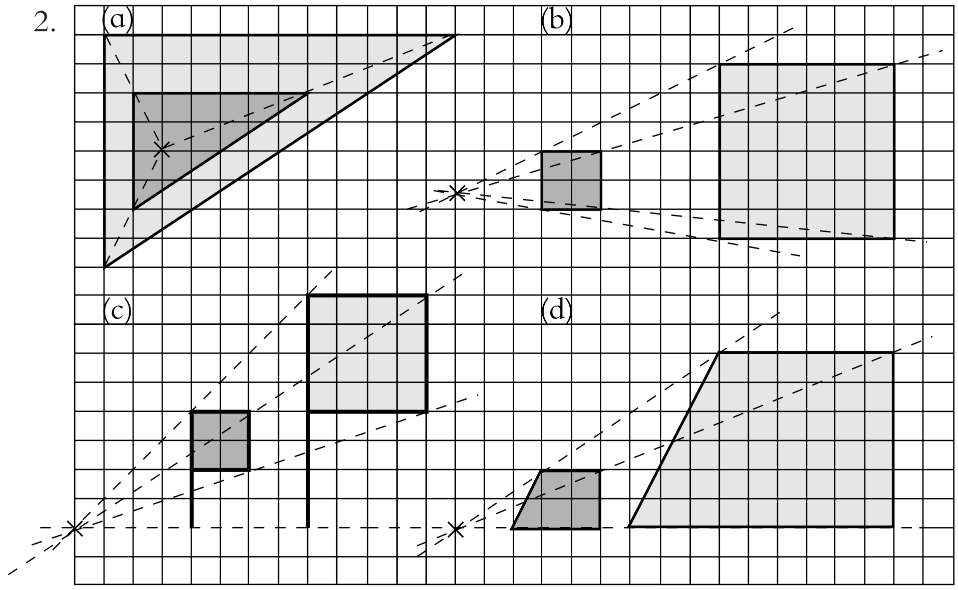
10.

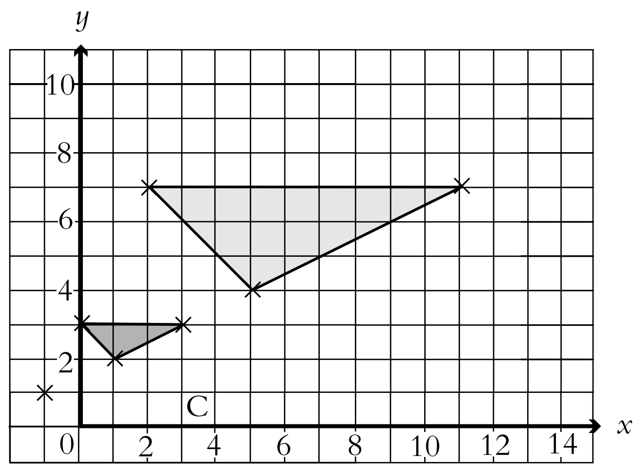


(Pg. 43) Skill Exercises: Finding the Centre of Enlargement

1.

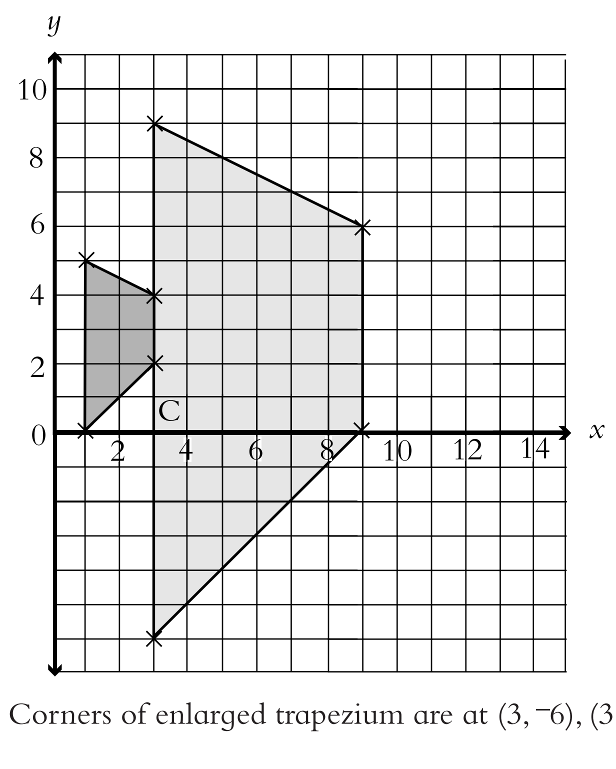


2. 

3. 

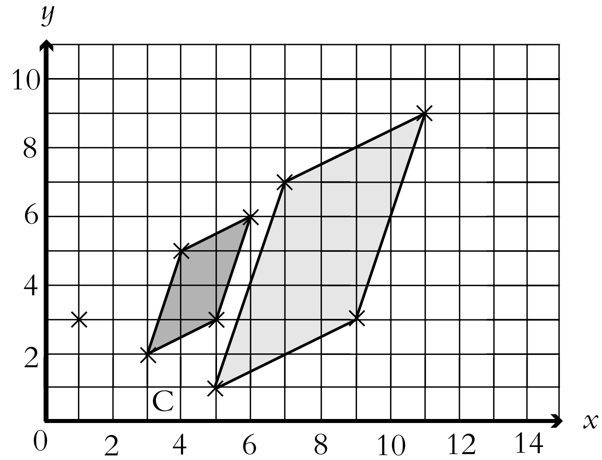
Scale factor = 3

Centre of enlargement is at (⁻1, 1)

4. 

Corners of enlarged trapezium are at (3, ⁻6), (3, 9), (9, 6) and (9, 0)

5.



6. (a) The scale factor for heights must be at most

$24 \div 6.5 = 3.692$ (to 3 d.p.),

the scale factor for widths must be at most $12 \div 4 = 3$,

so the maximum scale factor Lani can use is 3.

(b) The scale factor for heights must be at most $2.7 \div 6.5 = 0.415$ (to 3 d.p.),

the scale factor for widths must be at most $2.7 \div 4 = 0.675$,

so the maximum scale factor Lani can use is 0.415 (to 3 d.p.)

(c) The perimeter $= (\pi \times 6.6) + (2 \times 6.6)$

$= 33.93451151$ cm

$= 33.93$ cm (to 2 d.p.)

(Pg. 48) Skill Exercises: Recognising Similar Shapes

1. Scale factor $= \dfrac{30}{6} = 5$ CD $= 5 \times 16 = 80$ cm

2. Scale factor $= \dfrac{12}{6} = 2$

(a) AB $= 2 \times 2.5 = 5$ cm (b) EF $= \dfrac{13}{2} = 6.5$ cm

3. Scale factor = 8

(a) DE $= \dfrac{32}{8} = 4$ cm (b) AC $= 4 \times 8 = 32$ cm

(c) BC $= 3 \times 8 = 24$ cm

4. Scale factor = 7 GE = 42 cm

FG = 35 cm

5. (a) EG = 10 cm ($2 \times$ AC) (b) HJ = 30 cm ($6 \times$ AC)

 (c) EF = 12 cm ($\frac{1}{3}$ HI) (d) AB = 6 cm ($\frac{1}{6} \times$ HI)

6. (a) HI = 7.5 cm (b) BC = 3 cm

 (c) AC = 5 cm (d) DF = 7.5 cm

7. (a) IP = 8 cm (b) JK = 2 cm

 (c) LM = 2 cm (d) FG = 6 cm NO = 4 cm

 (e) EF = 4.5 cm

8. (a) (i) $\angle$ ABE = $\angle$ DBC (Vertically Opposite Angles)

 (ii) $\angle$ BAE = $\angle$ BDC (Alternate Angles)

 (iii) $\angle$ AEB = $\angle$ DCB (Alternate Angles)

 (b) AB = 16.4 cm; BE = 20 cm

9. (a) EB is parallel to DC.
 Angle AEB = angle ADC.
 Angle ABE = angle ACD.
 Angle EAB is common to both triangles.
 Therefore triangle ABE is similar to triangle ACD.

 or

 EB is parallel to DC.
 Triangle ABE has been enlarged to triangle ACD.
 A is the centre of enlargement.
 Scale factor is 1.5

 (b) AC = 6.6 cm; BC = 2.2 cm

 (c) AE = $13.5 \div 1.5$ DE = AD – AE

 = 9.0 cm = 13.5 – 9.0

 = 4.5 cm

10. (a) AC = $\frac{6}{10} \times 15 = 9$ cm

 CD = $\frac{8}{10} \times 15 = 12$ cm

 DE = 3 cm

(b) $GC = CB \times \frac{8}{6}$ $\qquad FC = CB \times \frac{10}{6}$

$= 10.8 \times \frac{8}{6}$ $\qquad = 10.8 \times \frac{10}{6}$

$= 14.4$ cm $\qquad = 18$ cm

$FG = FC - CB$

$= 18 - 14.4$

$= 3.6$ cm

(Pg. 56) Skill Exercises: Working with Line, Area and Volume Ratios

1. (a) 12 cm^2, 192 cm^2
   (b) 4
   (c) Area is 16 (4^2) times bigger

2. (a) $12 \times 2^2 = 48$ cm^2
   (b) $12 \times 3^3 = 108$ cm^2
   (c) $12 \times 6^2 = 432$ cm^2
   (d) $12 \times 10^2 = 1200$ cm^2

3.

Length of Sides Base	Height	Scale Factor	Area	Area Scale Factor
3 cm	4 cm	1	6 cm^2	1
6 cm	8 cm	2	24 cm^2	4
9 cm	12 cm	3	54 cm^2	9
12 cm	16 cm	4	96 cm^2	16
15 cm	20 cm	5	150 cm^2	25
18 cm	24 cm	6	216 cm^2	36
30 cm	40 cm	10	600 cm^2	100
4.5 cm	6 cm	1.5	13.5 cm^2	2.25

4. Area $= 25 \times 42$
   $= 1050$ cm^2

5. Area $= 9 \times 50$
   $= 450$ cm^2

6. $480 \div 30 = 16$ $\therefore$ The scale factor of enlargemetn is 4

7. (a) Smaller $2 \times 4 \times 3 = 24\text{ cm}^3$;
   Larger $4 \times 8 \times 6 = 192\text{ cm}^3$

   (b) Scale factor = 2

   (c) 8

   (d) $2^3 = 8$

8.

Dimensions			Scale Factor	Volume	Volume Factor
Width	Length	Height			
3 cm	6 cm	2 cm	1	36 cm^3	1
6 cm	12 cm	4 cm	2	288 cm^3	8
12 cm	24 cm	8 cm	4	2304 cm^3	64
15 cm	30 cm	10 cm	5	4500 cm^3	125
30 cm	60 cm	20 cm	10	36 000 cm^3	1000

9. Vol $= (3)^3 \times 32$
   $= 27 \times 32$
   $= 864\text{ cm}^3$

10. Scale factor = 2.5
    Vol $= (2.5)^3 \times 42$
    $= 656.25\text{ cm}^3$

(Pg. 60) Skill Exercises: Using Enlargement with Maps and Scale Models

1. (a) 400 cm = 4 m (b) 50 000 cm^2 = 5 m^2

   (c) 3.2 m^3 (3 200 000 cm^3)

2. (a) $50 \times 50\,000^2 = 125\,000\,000\,000\text{ cm}^2$

   (b) 12 500 000 m^2

   (c) 12.5 km^2 (1 km^2 = 1 000 000 m^2)

3. 7776 m^3

4. (a) $(2 \times 40\,000) \times (5 \times 40\,000) = 16\,000\,000\,000\text{ cm}^2$

   (b) 1 600 000 m^2

   (c) 1.6 km^2

5. (a) $20 \times 30 = 600$ cm
$= 6$ m

(b) $850 \times 30^2 = 765\ 000\ \text{cm}^2$
$= 76.5\ \text{m}^2$

(c) $144\ 400 \times 30^3 = 3\ 898\ 800\ 000$
$= 3898.8\ \text{m}^3$

6. (a) 60 cm : 120 cm $=$ 1 : 20

(b) $3015 \times 20^2 = 24\ 120\ 000\ \text{cm}^3$
$= 24.12\ \text{m}^3$

(c) $2 \times 20^2 = 800\ \text{cm}^2$
$= 0.08\ \text{m}^2$

7. $4 \times (200\ 000)^2 = 160\ 000\ 000\ 000\ \text{cm}^2$
$= 16\ 000\ 000\ \text{m}^2$
$= 16\ \text{km}^2$

8. $(50 \times 1\ 000\ 000 \times 10\ 000) \div 40\ 000^2$
convert to m^2 then $\text{cm}^2 \div (\text{scale})^2$
$= 312.5\ \text{cm}^2$

9. (a) 5000

(b) $(384 \times 10\ 000) \div 80^2 = 600\ \text{cm}^2$

(c) $(3840 \times 1\ 000\ 000) \div 80^3 = 7500\ \text{cm}^3$

10. $1\ \text{km}^2 = 1\ 000\ 000\ \text{m}^2$ and $1\ \text{m}^2 = 10\ 000\ \text{cm}^2$

$\therefore\ (5 \times 1\ 000\ 000 \times 10\ 000) \div (\text{scale})^2 = 20$

$(5 \times 10^{10}) \div (\text{scale})^2 = 20$

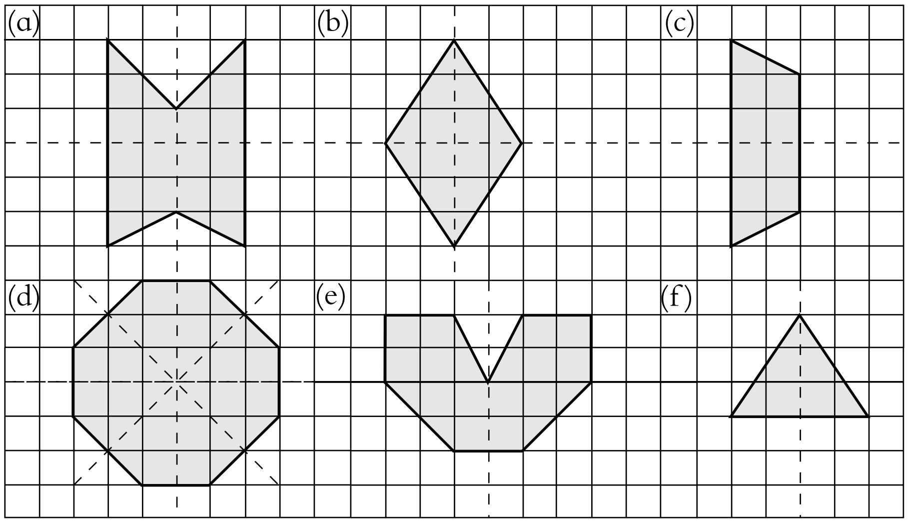
$\sqrt{5 \times 10^{10} \div 20} = \text{scale}$

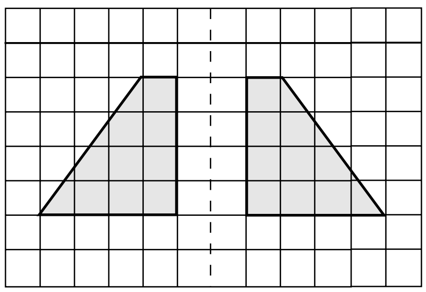
$\text{scale} = 50\ 000$

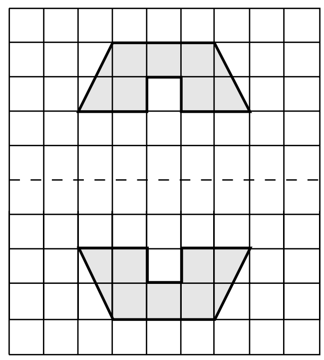
$\therefore$ map scale $= 1 : 50\ 000$

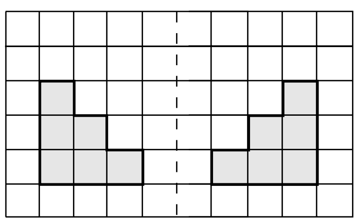
Section 7.2 Exploring Pattern Using Line And Rotational Symmetry

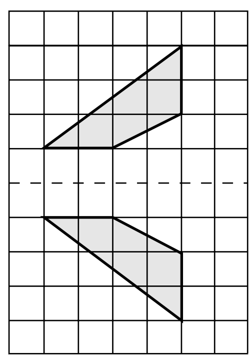
(Pg. 66) Skill Exercises: Working with Line Symmetry and Reflection

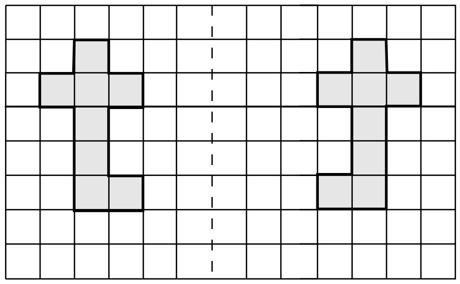
1.


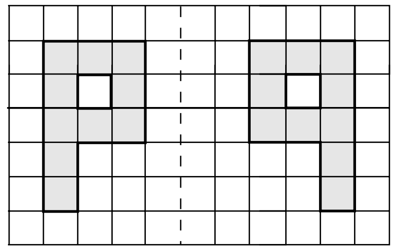
2. (a)


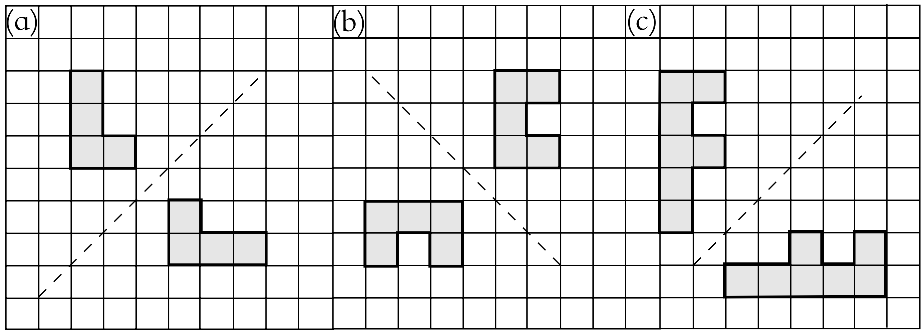
(b)


(c)


(d)


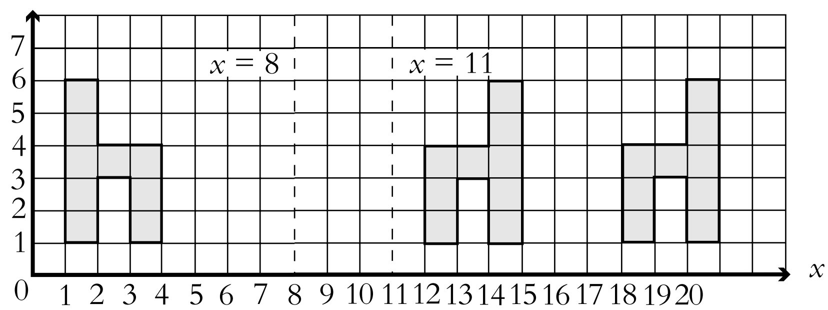
(e)


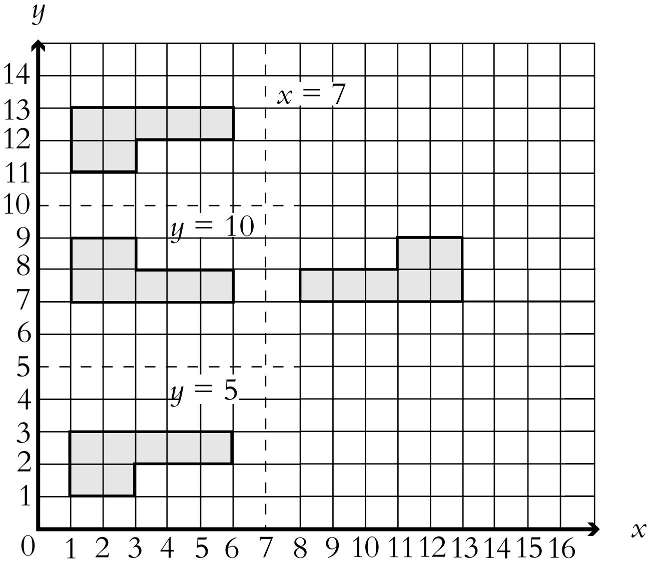
(f)


3. (a) (b) (c)


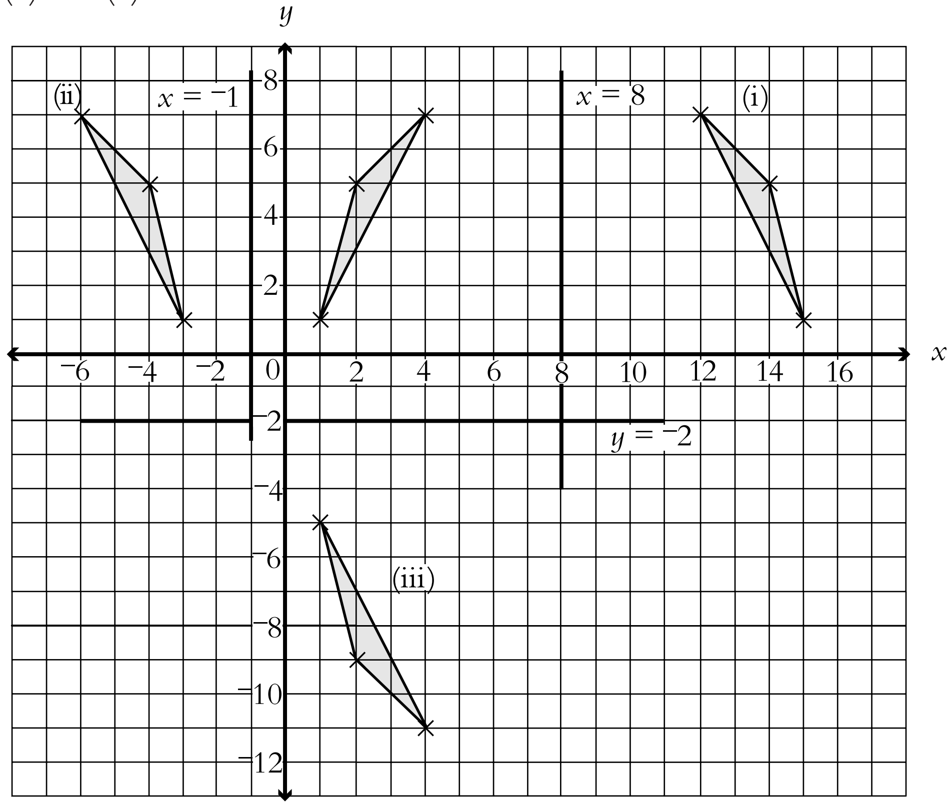
4 (a) and (b)

Reflection in the line $x = 8$ Reflection in the line $x = 11$



5.


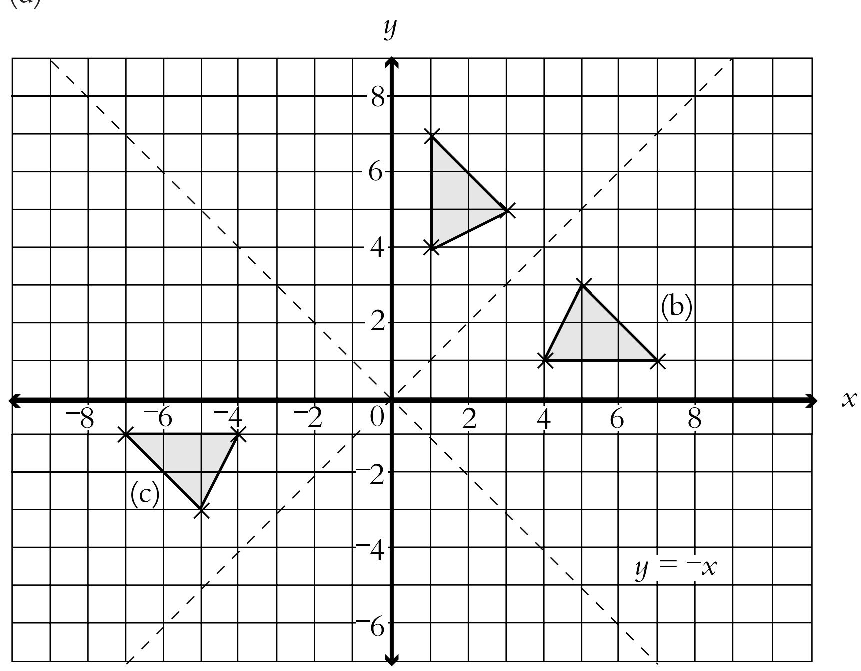
6. (a) and (b)



7. (a) $x = 5$ (b) $x = 9$ (c) $x = 10$

(d) $x = 16$ (e) $x = 21$ (f) $x = 14$

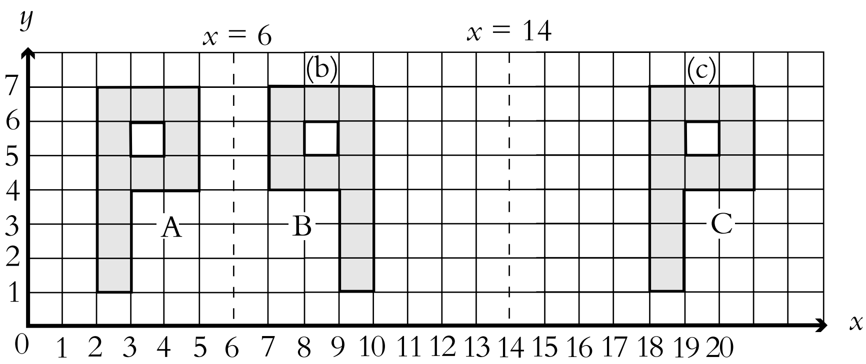
8. (a)



(b) Co-ordinates (4, 1) (7, 1) (5, 3)

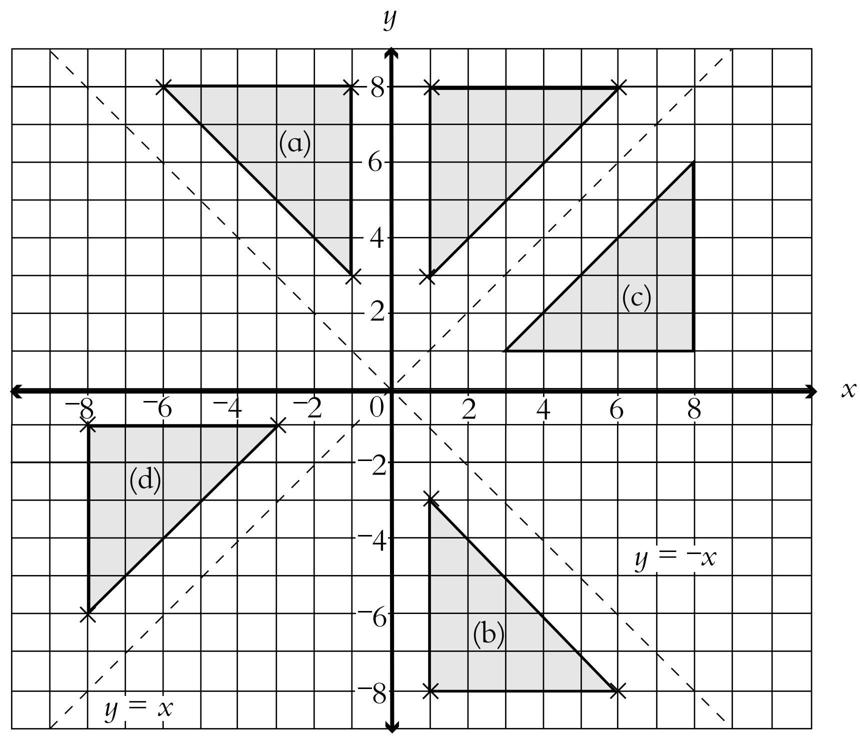
(c) Co-ordinates ($^-4$, $^-1$) ($^-7$, $^-1$) ($^-5$, $^-3$)

9. (a)

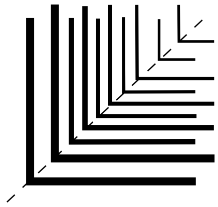


(d) Translation $\begin{bmatrix} 16 \\ 0 \end{bmatrix}$

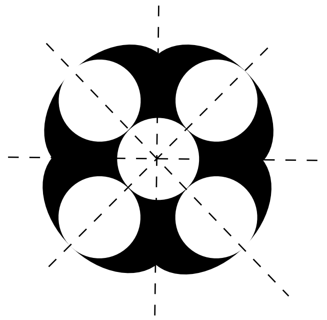
10.



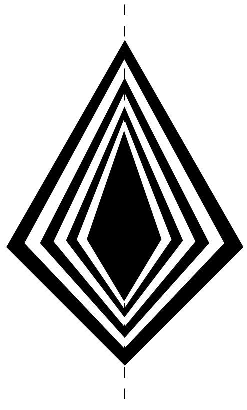
11. (a)



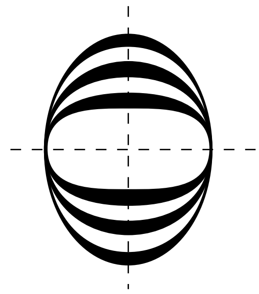
(b)



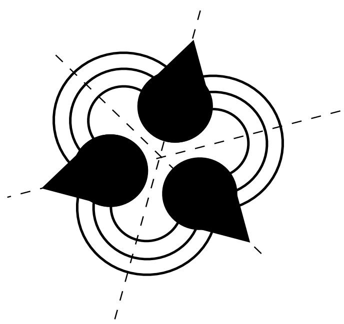
(c)



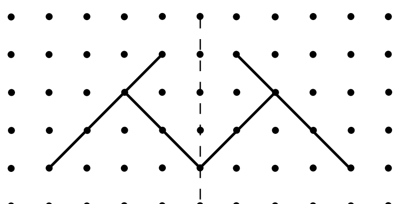
(d)



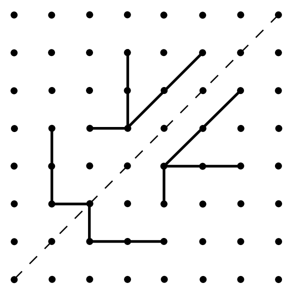
(e)



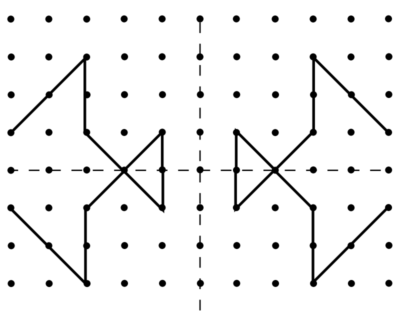
12. (a)



(b)



(c)



13. (a) $x = y$ (b) $14\frac{1}{2}$

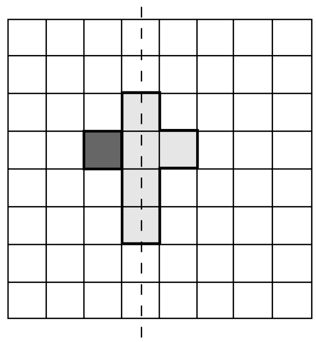
(c) (10, 12) is above the line because its y co-ordinate is greater than its x co-ordinate

(d) Any x co-ordinate less than 15, e.g. (13, 15)

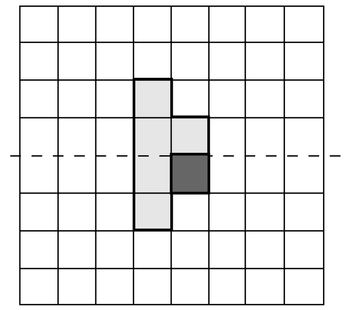
(e) Co-ordinates reversed

(f) (13, 20)

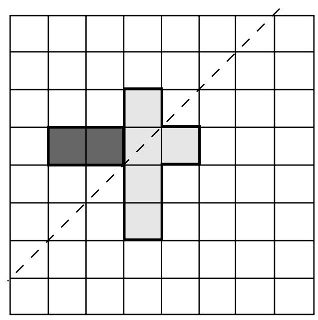
14. (a)



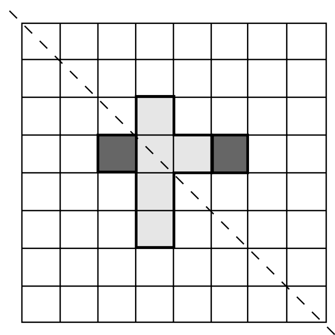
(b)



(c)



(d)



(Pg. 76) Skill Exercises: Rotating Shapes and Rotational Symmetry

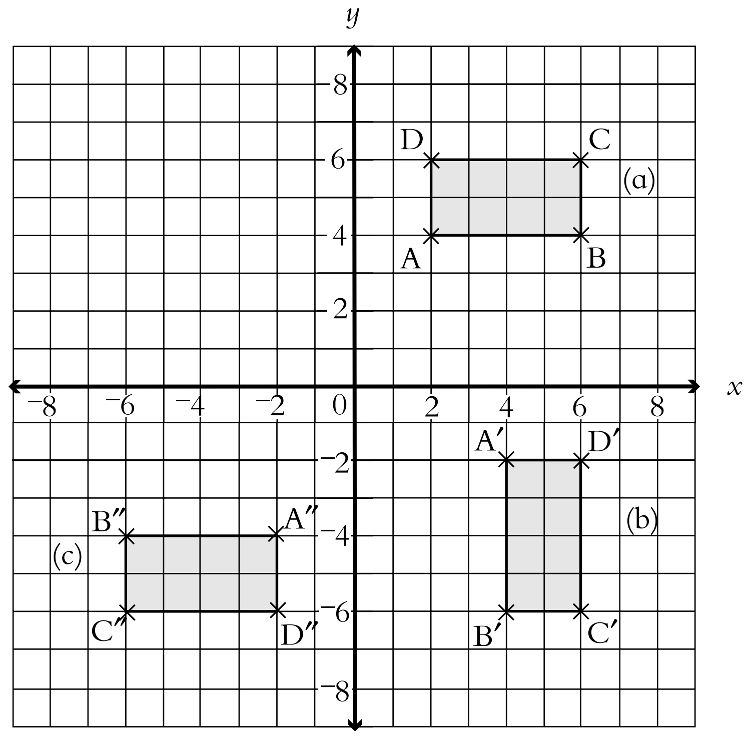
1. (a) 2 (b) 4

(c) 1 (d) 3

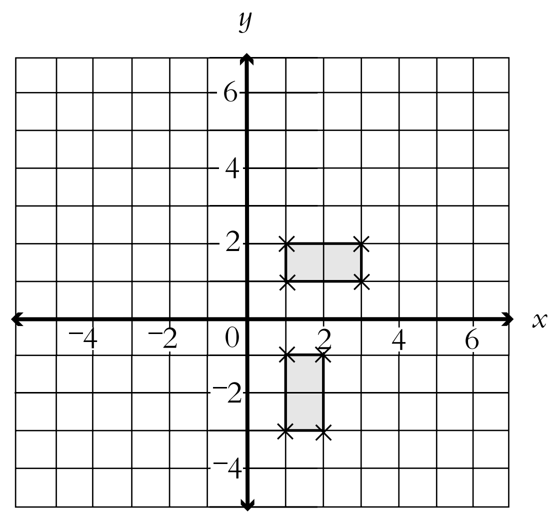
(e) 4 (f) 6

2. H, I, N, O, S, X, Z (depending on how the letters are printed or written)

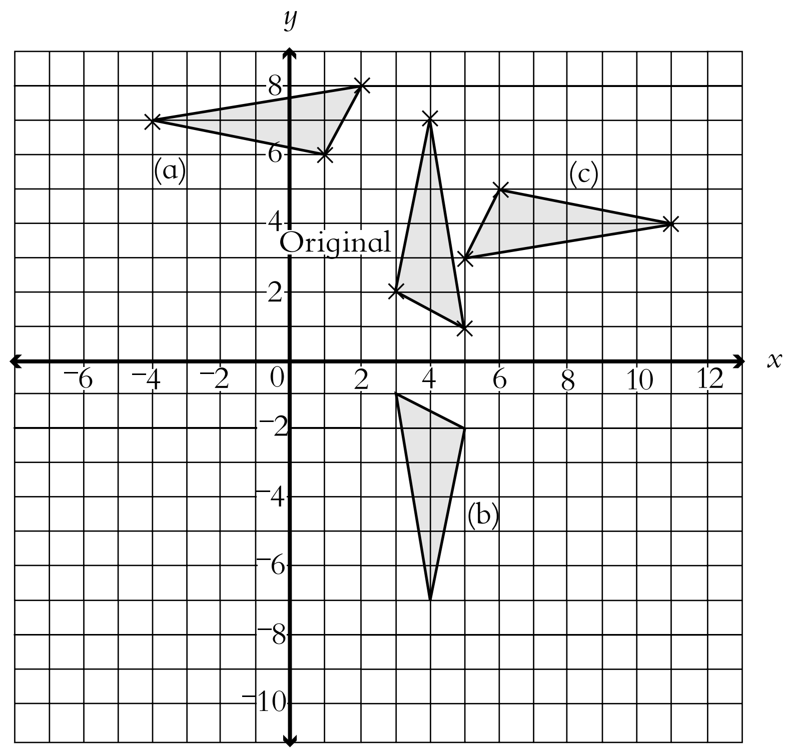
3.



4.



5.



(a) (1, 6), (2, 8), (−4, 7)

(b) (3, −1), 4, −7), (5, −2)

(c) (5, 3), (11, 4), (6, 5)

6. (a) Rotation through 90° clockwise about the origin (0, 0)

   (b) Rotation through 180° about the origin (0, 0)

   (c) Rotation through 90° anticlockwise about the origin (0, 0)

   (d) Rotation through 180° about (8, 0)

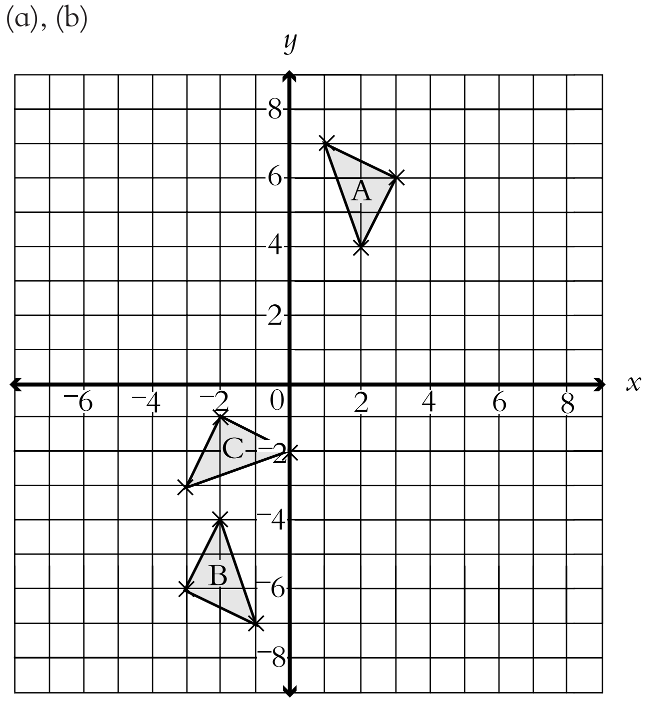
7. (a) Rotation through 90° clockwise about (4, 1)

   (b) Rotation through 90° clockwise about the origin (0, 0)

   (c) Rotation through 180° about the origin (0, 0)

   (d) Rotation through 90° anticlockwise about (0, 1)

8. (a), (b)

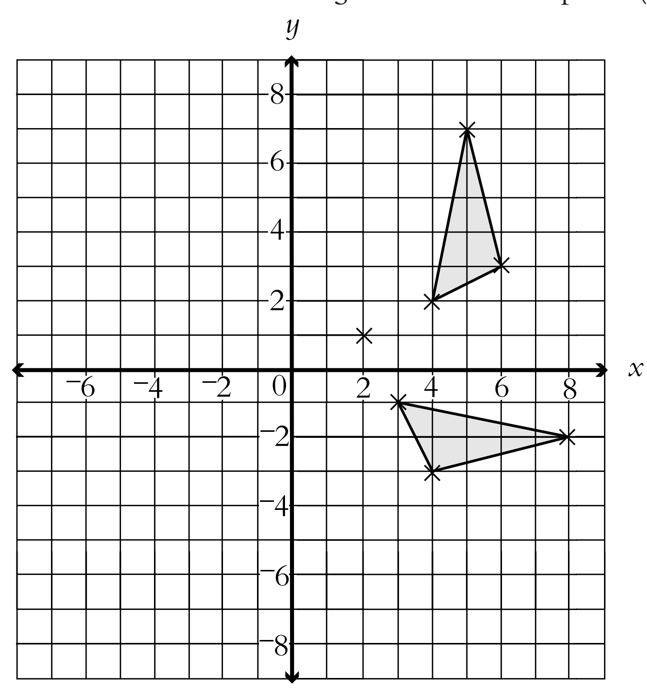


(c) Corners of triangle C have co-ordinates ($^-3$, $^-3$), ($^-2$, $^-1$), (0, $^-2$)

9. (a) (6, 7)

(b) (1.5, 7.5)

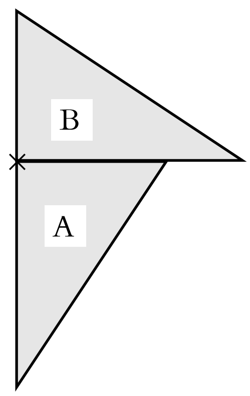
10. Rotation clockwise through 90° about the point (2, 1) (as diagram)



11. Lines of Symmetry

		0	1	2	3
Order of	1	E	F		
Rotational	2	B		C	
Symmetry	3	D			A

12. (a)



(b) 90°

13. (a) B1 Rotate 90° clockwise and then rotate 90° clockwise again

B2 Reflect vertical

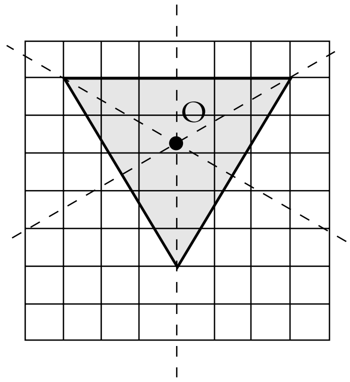
(b) A2 Rotate 90° clockwise and then rotate 90° clockwise again

B1 Reflect vertical and then rotate 90° clockwise

B2 Rotate 90° clockwise

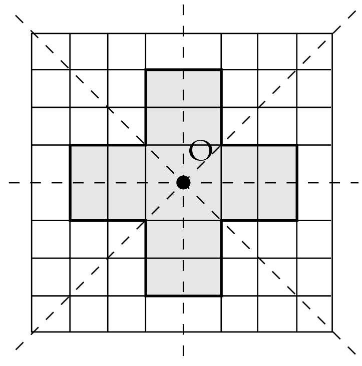
(Pg. 84) Skill Exercises: Examining the Symmetry of Polygons

1. (a)



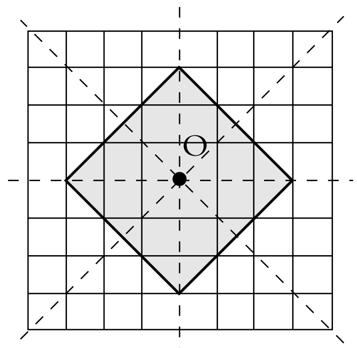
Order of rotational symmetry 3

(b)



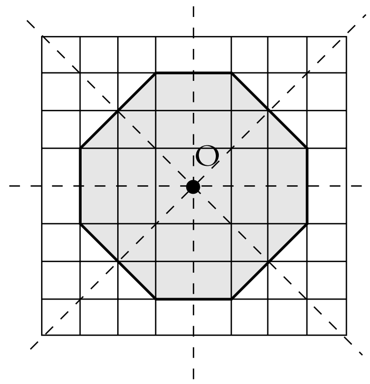
Order of rotational symmetry 4

(c)



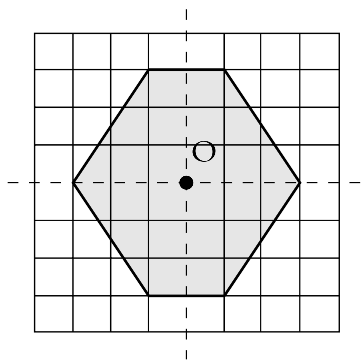
Order of rotational symmetry 4

(d)



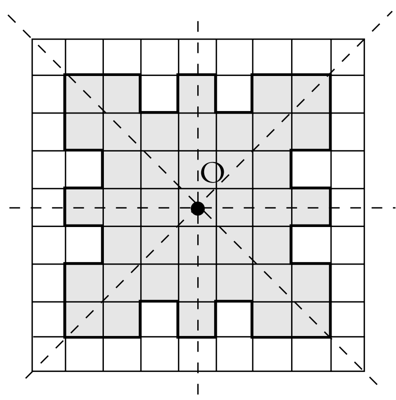
Order of rotational symmetry 4

(e)



Order of rotational symmetry 2

(f)



Order of rotational symmetry 4

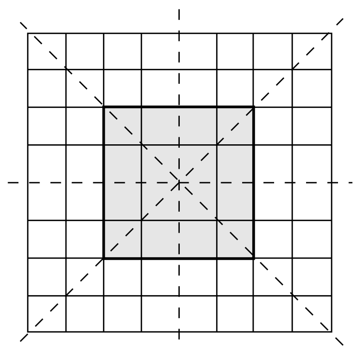
2.

	Order of Rotational Symmetry	No. Lines of Symmetry
(a)	4	4
(b)	1	1
(c)	2	2
(d)	2	2
(e)	2	2
(f)	1	1

3.	Order of Rotational Symmetry	No. of Lines of Symmetry
(a)	6	0
(b)	3	3

4.	Order of Rotational Symmetry	No. of Lines of Symmetry
Equilateral	3	3
Isosceles	1	1
Scalene	1	0

5. (a) 4 (b) 4



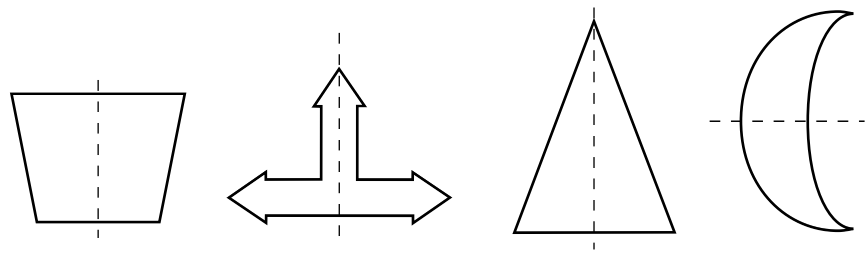
6. (a)

Shape	Order of Rotational Symmetry	Number of Lines of Symmetry
Equilateral triangle	3	3
Square	4	4
Regular pentagon	5	5
Regular hexagon	6	6
Regular heptagon (7 sides)	7	7
Regular octagon	8	8
Regular nonagon (9 sides)	9	9
Regular decagon (10 sides)	10	10
Regular dodecagon (12 sides)	12	12

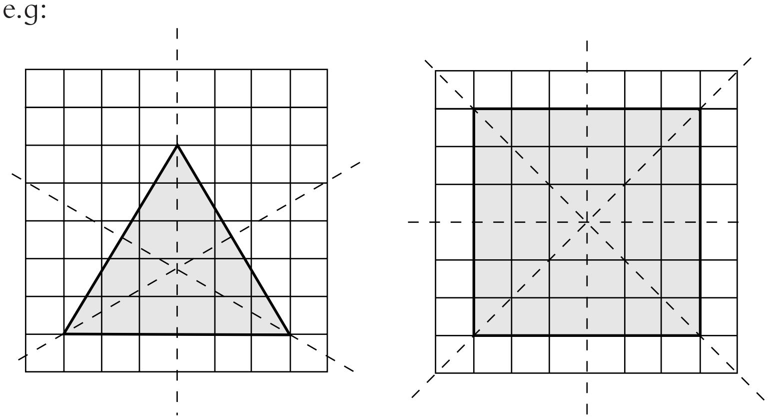
(b) The number of lines of symmetry = order of rotational symmetry for a regular polygon.

7. Own design.

8. Own design. e.g.



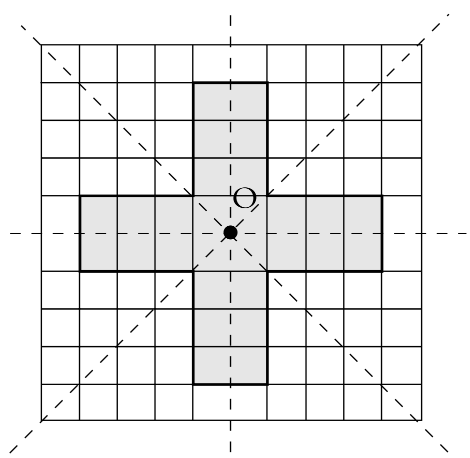
9. e.g:



Odd number of sides – lines of symmetry through vertex and middle of opposite side.

Even number of sides – lines of symmetry through middle of opposite sides, or diagonal vertices.

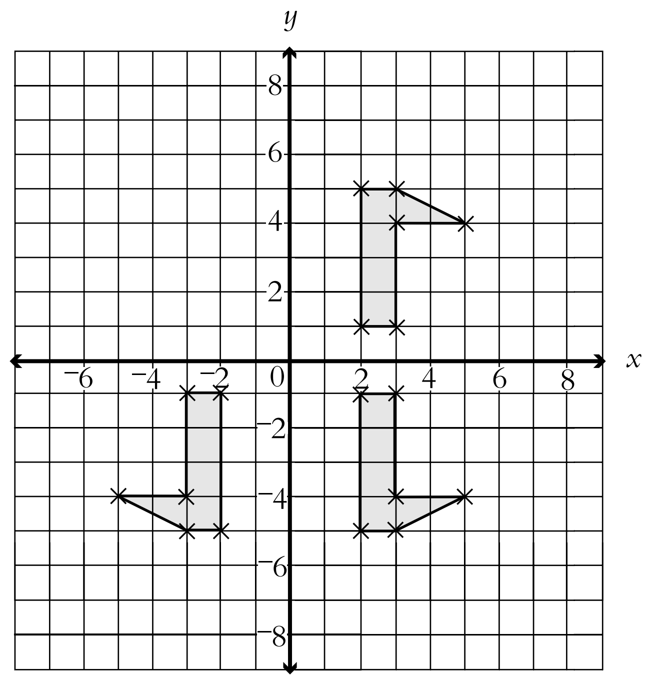
10.



Order of rotational symmetry = 4

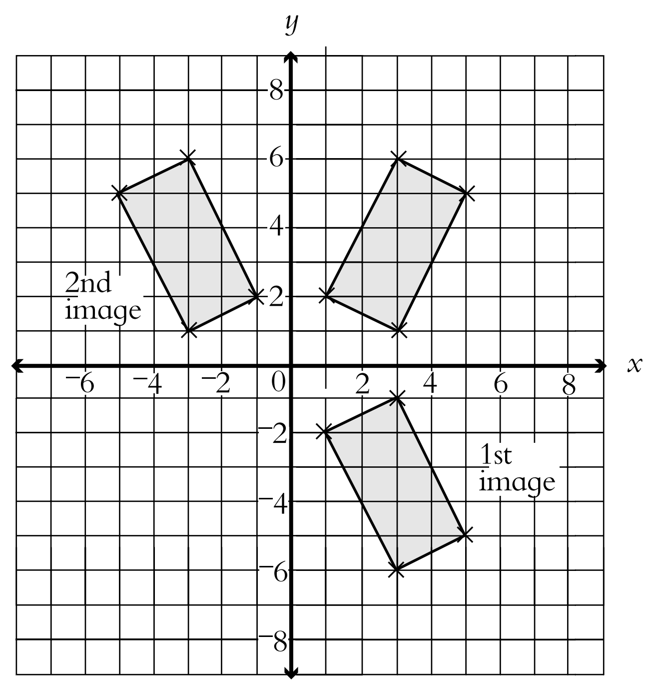
(Pg. 89) Skill Exercises: Combining Transformations

1. (a)



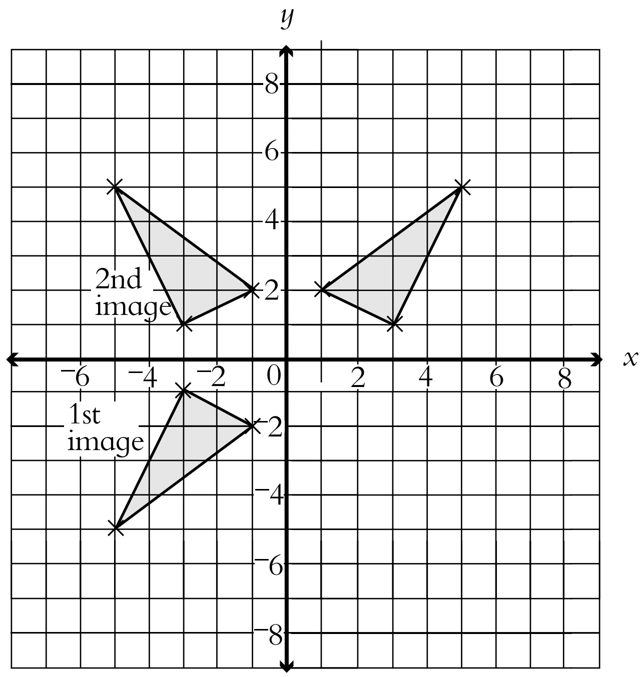
(b) Rotation of 180° about the origin

2. (a)



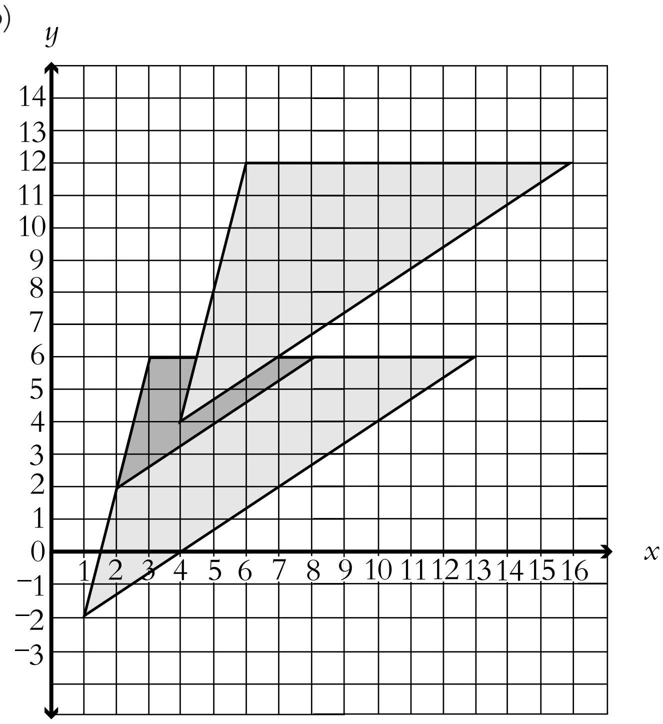
(b) The single transformation is a reflection in the y-axis

3. (a)



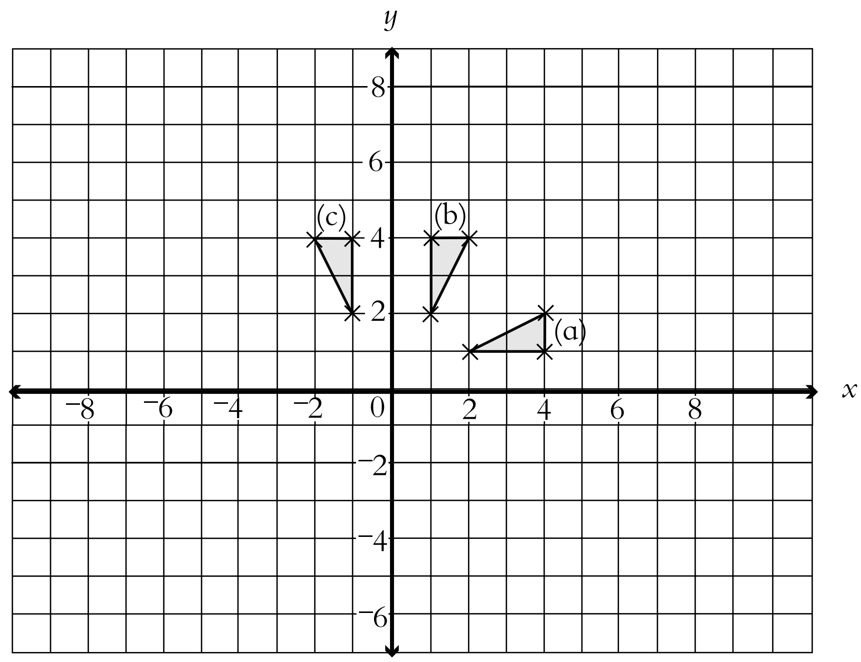
(b) The single transformation is a reflection is the y-axis

4. (a), (b)



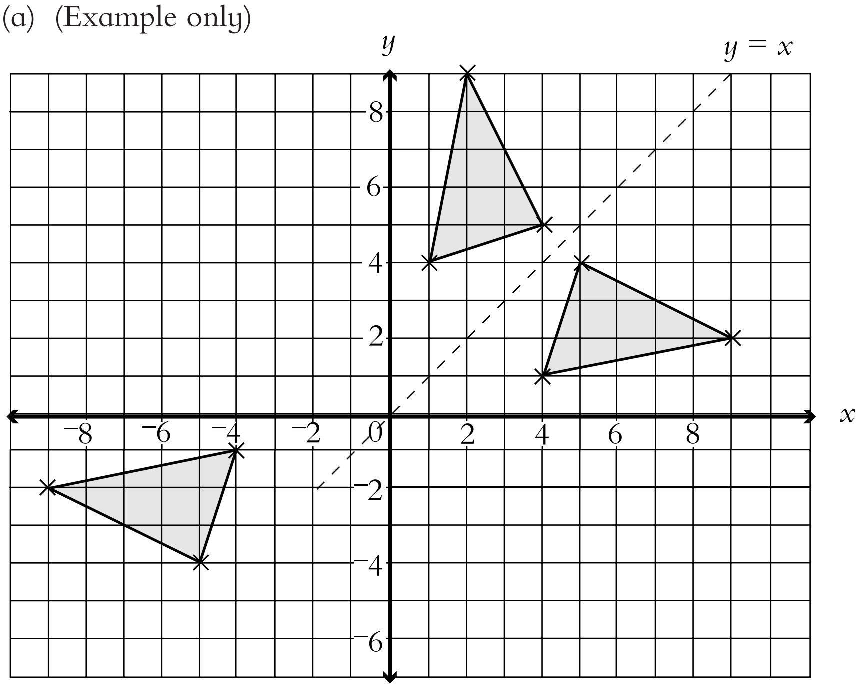
(c) Enlargement centre (3, 6), scale factor 2

5.



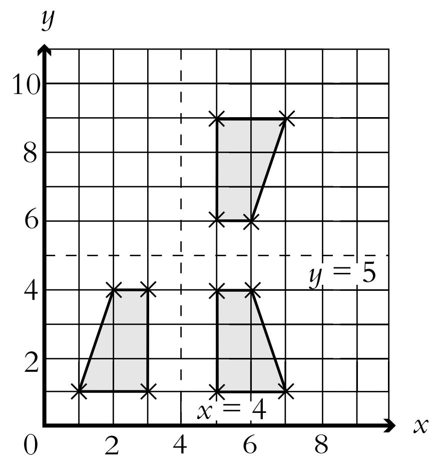
(d) 90° anticlockwise rotation about the origin

6. (a) (Example only)



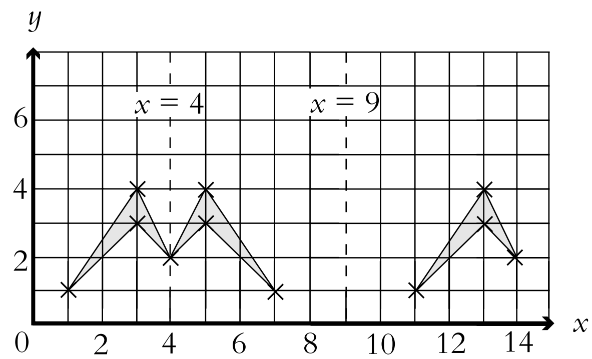
(b) 180° anticlockwise rotation about the origin

7. (a)



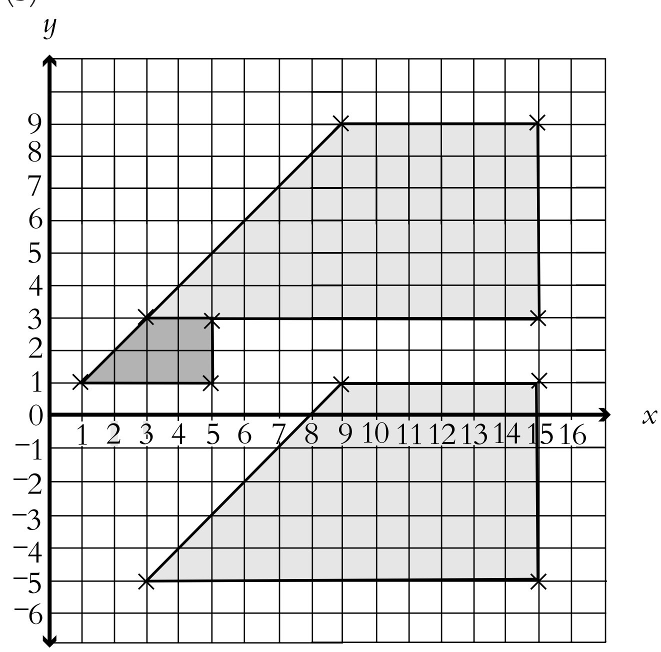
(b) 180° rotation about $(4, 5)$

8. (a)



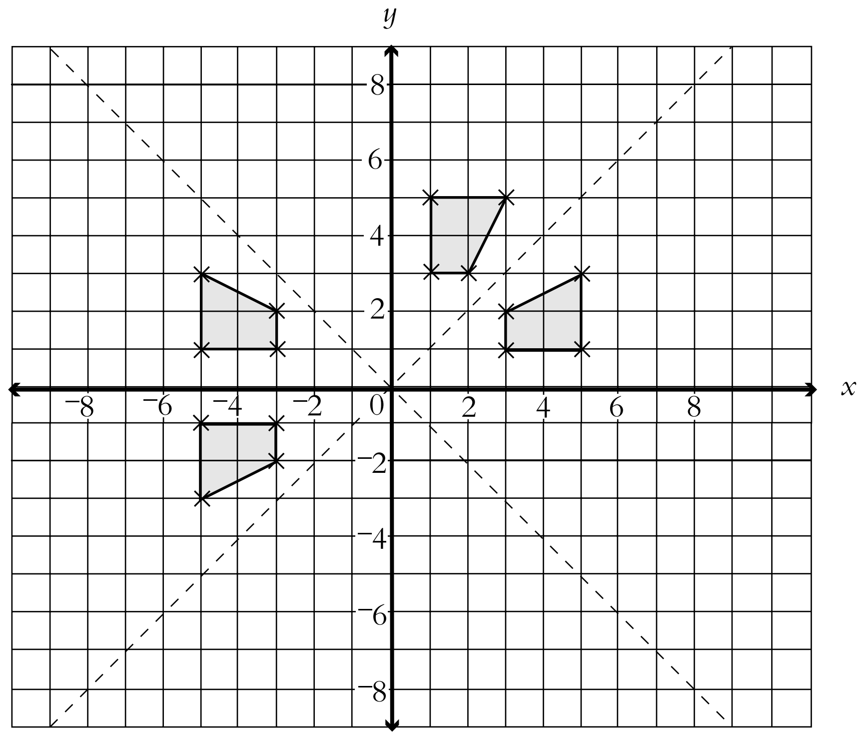
(b) There are many ways of doing this; one of the easiest is to reflect the original shape in the line $x = 4$ and then to reflect its image in the line $x = 9$, as in the diagram above.

9. (a), (b)



(c) Enlargement scale factor 3, centre $(0, 0)$

10. Single transformation is a reflection in the y-axis.



11. Possible answers are:

(i) rotate 90° anticlockwise about (3, 2) then reflect in the line $x = 5$

(ii) reflect in the line $y = 0$ then rotate 90° anticlockwise about (3, 2)

(iii) rotate 90° anticlockwise about (3, ⁻2) then reflect in the line $y = 0$

(iv) reflect in the line $x = 1$ then rotate through 90° clockwise about (k, $k - 5$) (3, ⁻2)