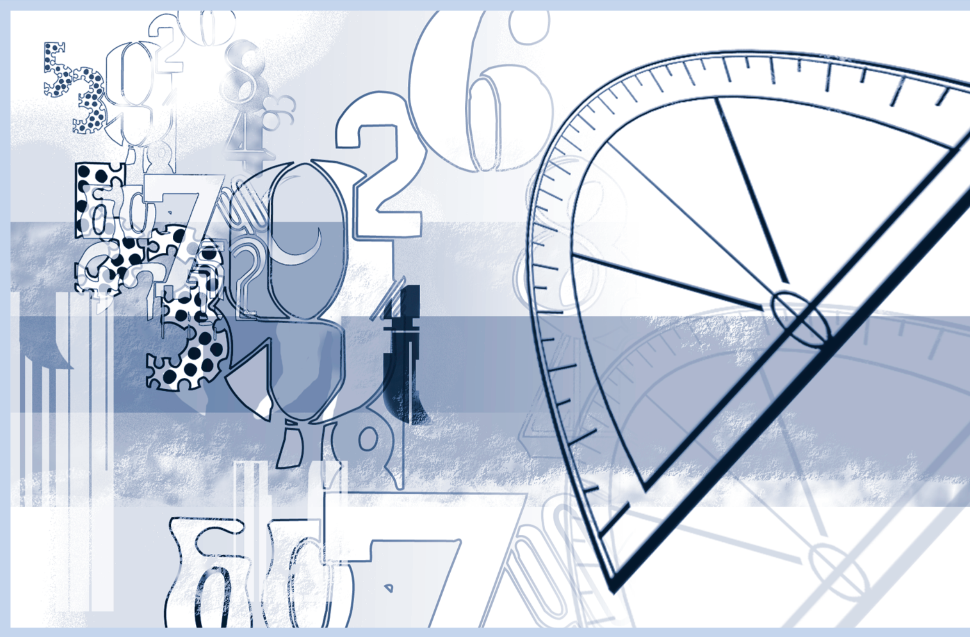
Book 2

Year 11



Mathematics

Mathematics

Year 11 Book Two

GOVERNMENT OF SĀMOA
MINISTRY OF EDUCATION, SPORTS AND CULTURE

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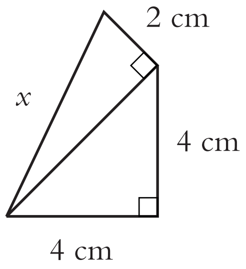
Section 5.1 Solving Problems With 2-Dimensional Figures

Working with Pythagoras' Theorem

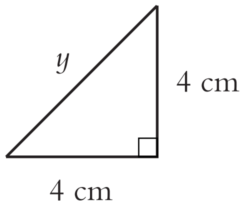
Example 1

Find the length of the side marked x in the diagram.



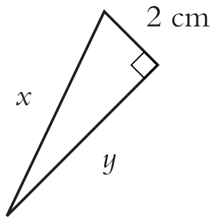
Solution

First consider the lower triangle. The unknown length of the hypotenuse has been marked y.



$$\begin{aligned} y^2 &= 4^2 + 4^2 \\ y^2 &= 16 + 16 \\ y^2 &= 32 \text{ cm} \end{aligned}$$

The upper triangle can now be considered, using the value for y^2. From the triangle, $x^2 = y^2 + 2^2$, and using $y^2 = 32$ gives

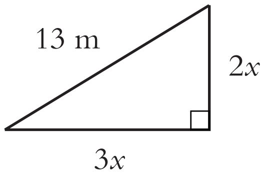


$$\begin{aligned} x^2 &= 32 + 4 \\ x &= \sqrt{36} \\ x &= 6 \text{ cm} \end{aligned}$$

Note: When finding the side x, it is not necessary to find $\sqrt{32}$, but to simply use $y^2 = 32$.

Example 2

Find the value of x as shown on the diagram, and state the lengths of the two unknown sides.



Solution

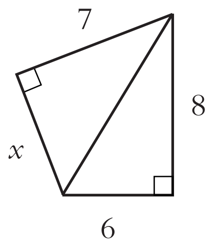
Using Pythagoras' Theorem gives

$$\begin{aligned} 13^2 &= (2x)^2 + (3x)^2 \\ 169 &= 4x^2 + 9x^2 && \text{(since } (2x)^2 = 2^2x^2 = 4x^2 \text{)} \\ 169 &= 13x^2 \\ 13 &= x^2 \\ x &= \sqrt{13} \\ &= 3.61 \text{ cm} && \text{(to 2 decimal places)} \end{aligned}$$

Skill Exercises: Working with Pythagoras' Theorem

1. Find the length of the side marked x in each diagram.

(a)



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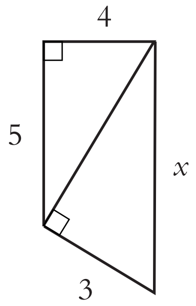

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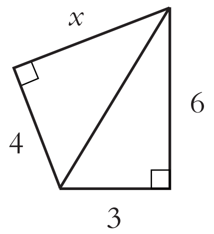

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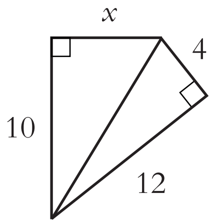

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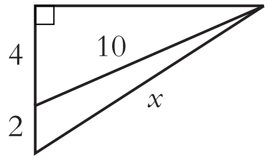

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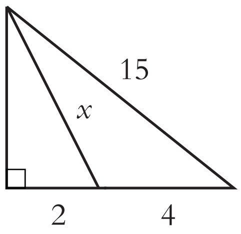
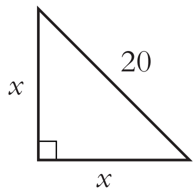


2. Find the length of the side marked x in the following situations.

(a)



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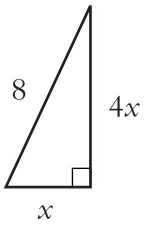

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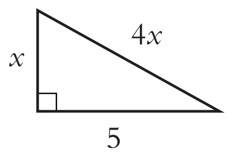

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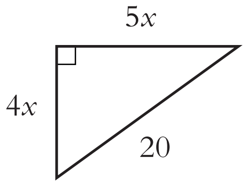
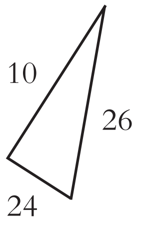


3. Which of the following triangles are right-angled triangles?

(a)



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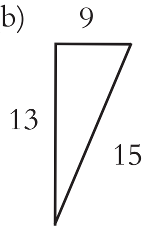

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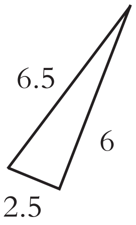

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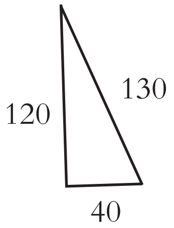
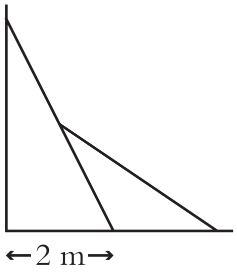


4. A ladder of length 4 metres leans against a vertical wall. The foot of the ladder is 2 metres from the wall. A plank that has a length of 5 metres rests on the ladder, so that one end is halfway up the ladder.

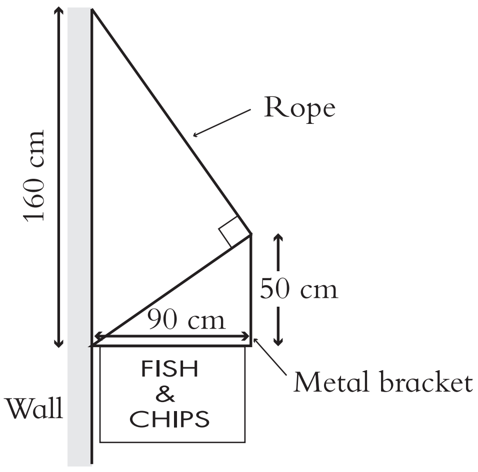
   (a) How high is the top of the ladder?

   (b) How high is the top of the plank?

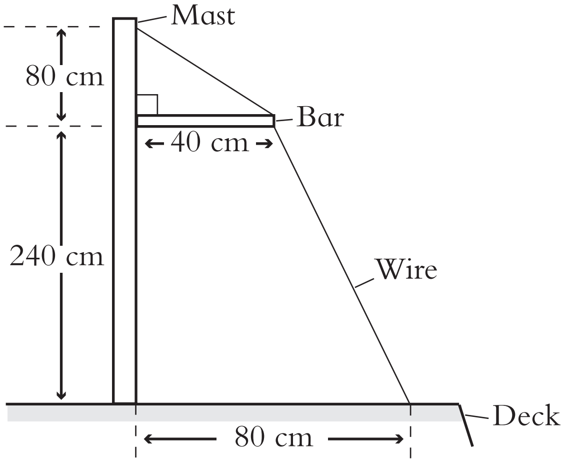
   (c) How far is the bottom of the plank from the wall?



5. The diagram shows how the sign that hangs over a Fish and Chip shop is suspended by a rope and a triangular metal bracket. Find the length of the rope.

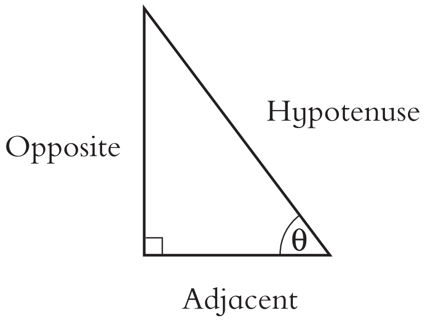


6. The diagram shows how a wire is attached to the mast of a sailing boat. A bar pushes the wire out away from the mast. Find the total length of the wire.



Finding Lengths in Right-Angled Triangles

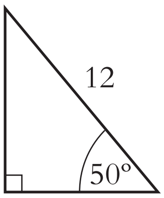
When working in a right-angled triangle, the longest side is known as the *hypotenuse*. The other two sides are known as the *opposite* and the *adjacent*. The adjacent is the side next to a marked angle, and the opposite side is opposite this angle.



For a right-angled triangle, the *sine*, *cosine* and *tangent* of the angle θ are defined as:

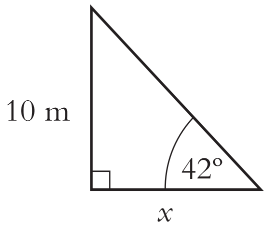
$$\boxed{\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}} \quad \boxed{\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}} \quad \boxed{\tan\theta = \frac{\text{opposite}}{\text{adjacent}}}$$

When one angle and the length of one side are known, it is possible to find the lengths of other sides in the same triangle, by using sine, cosine or tangent.



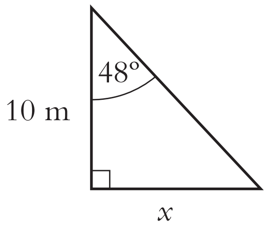
Example 1

Find the length marked x in the triangle.



Solution

This problem will involve tangent, so use the other angle which is 90°– 42° = 48°, so that x is the opposite.



Then $\quad \text{opposite} = x$

$\text{adjacent} = 10$ metres

And using $\quad \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Gives $\quad \tan 48° = \dfrac{x}{10}$

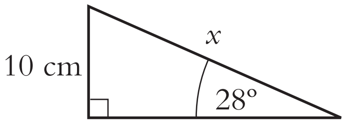
Multiplying both sides by 10 gives

$$x = 10 \times \tan 48°$$

$$= 11.1 \text{ metres} \quad \text{(to 1 decimal place)}$$

Example 2

Find the length of the hypotenuse, marked x, in the triangle.



Solution

In this triangle, hypotenuse $= x$

opposite $=$ 10 cm

Use sine because it involves hypotenuse and opposite.

Using $$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Gives $$\sin 28^\circ = \frac{10}{x}$$

Where x is the length of the hypotenuse.

Multiplying both sides by x gives

$$x \sin 28^\circ = 10$$

Then dividing both sides by sin 28° gives

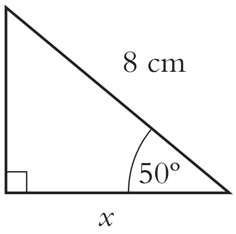
$$x = \frac{10}{\sin 28^\circ}$$

$$= 21.3 \text{ cm} \quad \text{(to 1 decimal place)}$$

Skill Exercises: Finding Lengths in Right-Angled Triangles

1. Find the length of the side marked x in each triangle.

(a)



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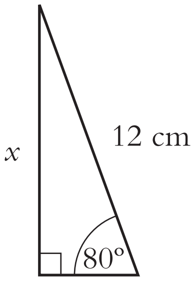

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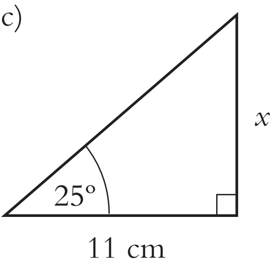

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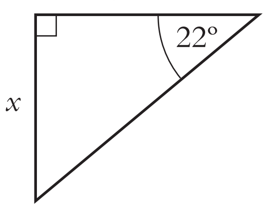

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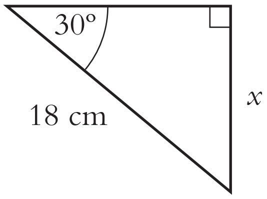

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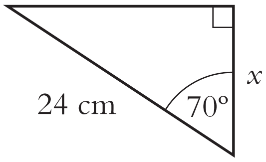
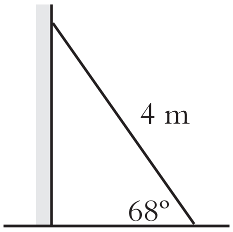
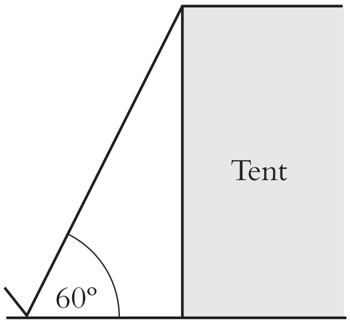


2. A ladder leans against a wall as shown in the diagram.
   (a) How far is the top of the ladder from the ground?
   (b) How far is the bottom of the ladder from the wall?

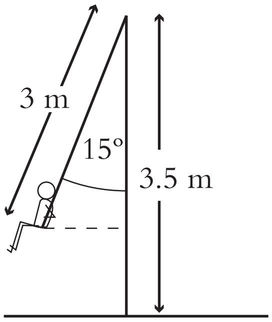


3. A rope is attached to a tent peg and the top of a tent pole so that the angle between the peg and the bottom of the pole is 60°.
   (a) Find the height of the pole if the peg is 1 metre from the bottom of the pole.
   (b) If the length of the rope is 1.4 metres, find the height of the pole.
   (c) Find the distance of the peg from the base of the pole if the length of the rope is 2 metres.



4. A child is on a swing. The highest position that she reaches is as shown.

   Find the height of the swing seat above the ground in this position.



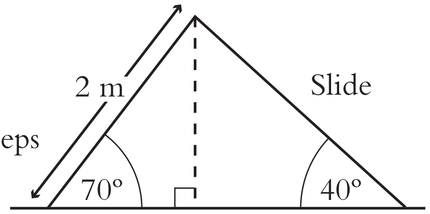
5. A laser beam shines on the side of a building. The side of the building is 500 metres from the source of the beam, which is at an angle of 16° above the horizontal. Find the height of the point where the beam hits the building.

6. A ship sails 400 km on a bearing of 075°.
   (a) How far east has the ship sailed?
   (b) How far north has the ship sailed?

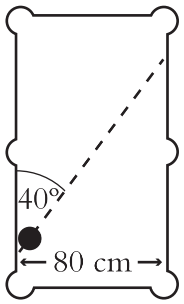
7. An aeroplane flies 120 km on a bearing of 210°.
   (a) How far south has the aeroplane flown?
   (b) How far west has the aeroplane flown?

8. A kite has a string of length 60 metres. On a windy day all the string is let out and makes an angle of between 20° and 36° with the ground. Find the minimum and the maximum heights of the kite.

9. The diagram shows a slide.
   (a) Find the height of the top of the slide.
   (b) Find the length of the slide.

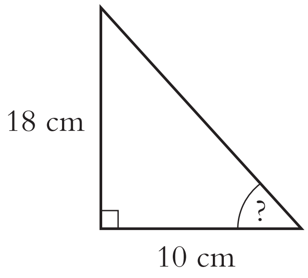


10. A snooker ball rests against the side cushion on a snooker table. It is hit so that it moves at 40° to the side of the table. How far does the ball travel before it hits the cushion on the other side of the table?



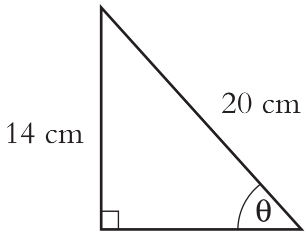
Finding Angles in Right-Angled Triangles

If the lengths of any two sides of a right angle triangle are known, then sine, cosine and tangent can be used to find the angles of the triangle.



Example 1

Find the angle marked θ in the triangle shown.



Solution

In this triangle, hypotenuse = 20 cm

opposite = 14 cm

Using $$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

gives $$\sin\theta = \frac{14}{20}$$

$$= 0.7$$

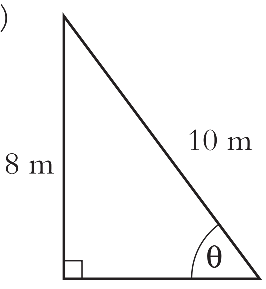
Then using the SHIFT and SIN buttons on a calculator gives:

$$\theta = 44.4° \text{ (to 1 d.p.)}$$

Skill Exercises: Finding Angles in Right-Angled Triangles

1. Find the angle θ of:

(a)



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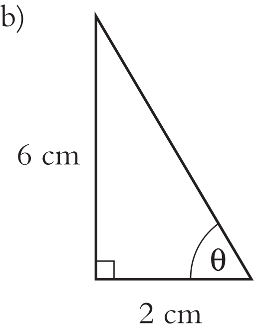

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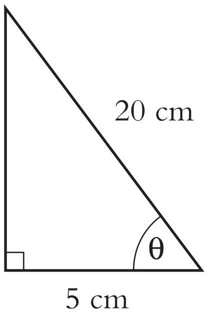

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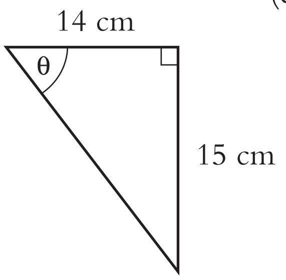

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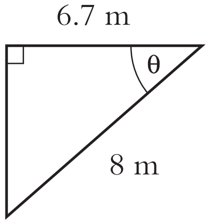

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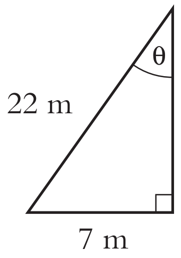
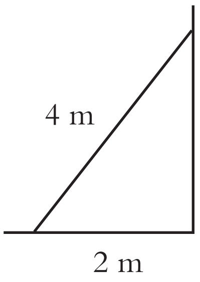
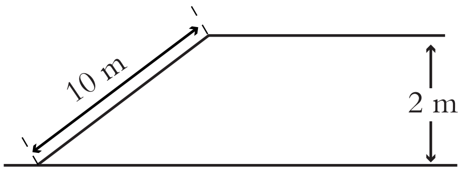


2. A ladder leans against a wall. The length of the ladder is 4 metres and the base is 2 metres from the wall. Find the angle between the ladder and the ground.



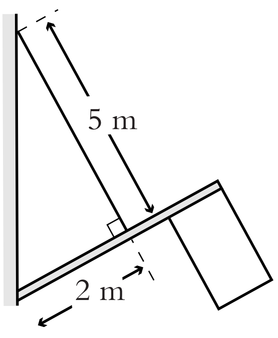
3. As cars drive up a ramp at a multi-storey car park, they go up 2 metres. The length of the ramp is 10 metres. Find the angle between the ramp and the horizontal.



4. A flag pole is fixed to a wall and supported by a rope, as shown. Find the angle between:

(a) the rope and the wall.

(b) the pole and the wall.

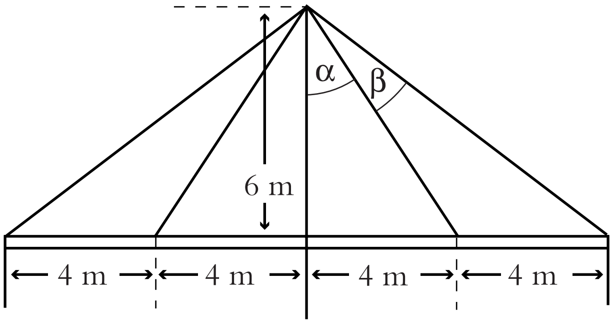


5. The mast on a yacht is supported by a number of wire ropes. One, which has a length of 15 metres, goes from the top of the mast at a height of 10 metres, to the front of the boat.
   (a) Find the angle between the wire rope and the mast.
   (b) Find the distance between the base of the mast and the front of the boat.

6. Siliko runs 500 metres east and 600 metres north. If he had run directly from his starting point and to his final position, what bearing should he have run on?

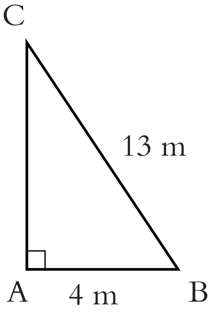
7. A ship is 50 km south and 70 km west of the harbour that it is heading for. What bearing should it sail on to reach the harbour?

8. The diagram shows a simple bridge, which is supported by four steel cables.
   (a) Find the angles at α and β.
   (b) Find the length of each cable.



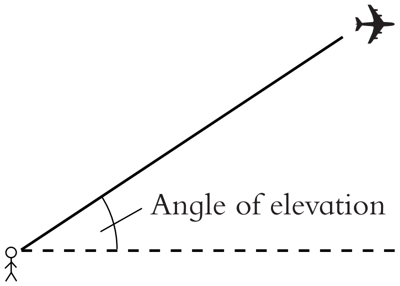
9. A rope has a length of 20 metres. When a boy hangs at the centre of the rope, the centre is 1 metre below its normal horizontal position. Find the angle between the rope and the horizontal in this position.

10. ABC is a right-angled triangle. AB is of length 4 metres and BC is of length 13 metres.
    (a) Calculate the length of AC.
    (b) Calculate the size of angle ABC.

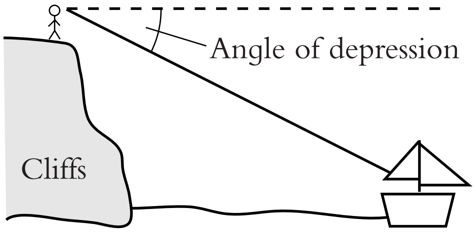


Solving Problems with Trigonometry

When you look *up* at something, such as an aeroplane, the angle between your line of sight and the horizontal is called the *angle of elevation*.



If you look *down* at something, then the angle between your line of sight and the horizontal is called the *angle of depression*.

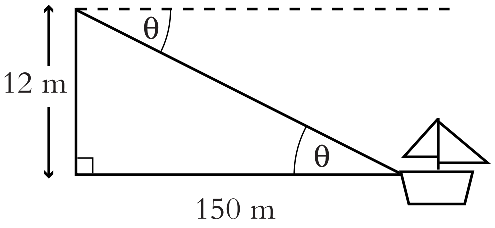


Example 1

A man looks out to sea from a cliff top at a height of 12 metres. He sees a boat that is 150 metres from the cliffs. What is the angle of depression?

Solution

The situation can be represented by the triangle shown in the diagram, where θ is the angle of depression.



In this triangle, opposite $=$ 12 m

adjacent $=$ 150 m

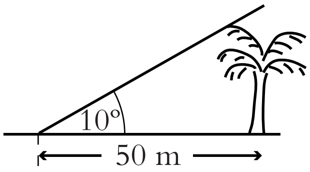
Using $$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

gives $$\tan\theta = \frac{12}{150}$$

Using a calculator gives $\theta = 4.6°$ (to 1 d.p.)

Skill Exercises: Solving Problems with Trigonometry

1. To find the height of a tree, some students walk 50 metres from the base of the tree and measure the angle of elevation as 10º. Find the height of the tree.



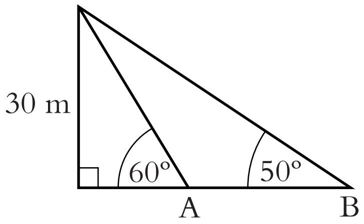
2. From the distance of 20 metres from its base, the angle of elevation of the top of a pylon is 32º. Find the height of the pylon.

3. The height of a church tower is 15 metres. A man looks at the tower from a distance of 120 metres. What is the angle of elevation of the top of the tower from the man?

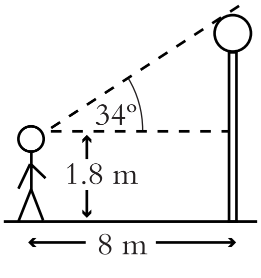
4. The harbour master at Apia looks out from his office at a height of 9 metres and sees a boat at a distance of 500 metres from the tower. What is the angle of depression of the boat from the tower?

5. A lighthouse is 20 metres high. A lifeboat is drifting and one of the people in it estimates the angle of elevation of the top of the lighthouse as 3º.
   (a) Use the estimated angle to find the distance of the lifeboat from the lighthouse.
   (b) If the lifeboat is in fact 600 metres from the lighthouse, find the correct angle of elevation.

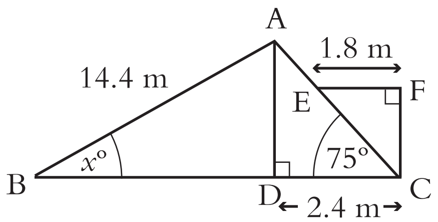
6. A radio mast is supported by two cables as shown. Find the distance between the two points A and B.



7. A man stands at a distance of 8 metres from the lamppost. When standing as shown, he measures the angle of elevation as 34º. Find the height of the lamppost.



8. The diagram represents a triangular roof frame ABC with a window frame EFC. BDC and EF are horizontal and AD and FC are vertical.
   (a) Calculate the height AD.
   (b) Calculate the size of the angle marked xº in the diagram.
   (c) Calculate FC.

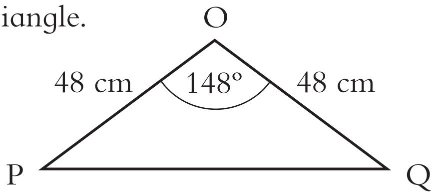


Dealing with Angles Larger than 90°

The sin, cosine and tangent ratios and Pythagoras' Theorem can only be used with right-angled triangles. If a problem involves a triangle with an angle that is larger then 90°, it may be possible to make a right-angled triangle.

Example 1

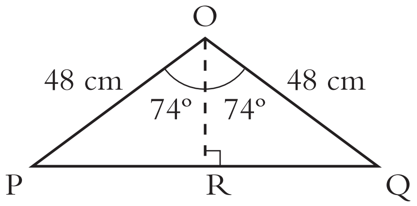
Find the length PQ of this isosceles triangle.



Solution

Draw a line from O so that it makes a right angle with PQ.

Find the distance PR.



$$\sin 74^\circ = \frac{PR}{48}$$

$$PR = 48 \times \sin 74^\circ$$

$$= 46.14 \text{ cm}$$

and $RQ = 46.14$ cm

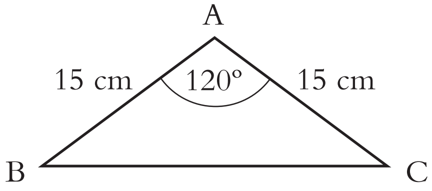
therefore $PQ = 46.14 \text{ cm} + 46.14 \text{ cm}$

$$= 92.28 \text{ cm}$$

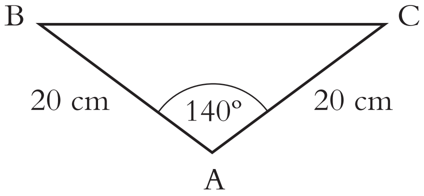
Skill Exercises: Dealing with Angles Larger than 90°

1. Find the length BC of these isosceles triangles.

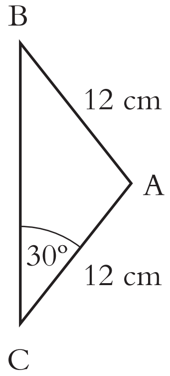
(a)



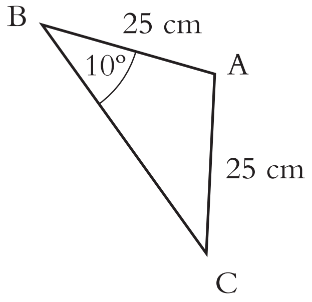
(b)



(c)



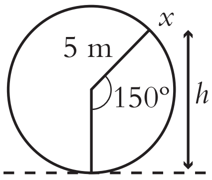
(d)

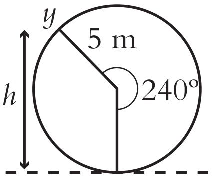


2. A ferris wheel has a radius of 5 cm and travels round in an anti-clockwise direction. Brian and Selena are on the wheel.

   (a) If they are at point x, what is their height above the ground?

   (b) If they are at point y, what is their height above the ground?

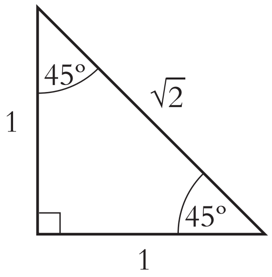


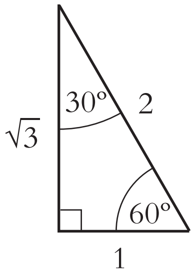


Section 5.2 Graphing Trigonometric Functions

Expressing Trigonometric Ratios as Fractions

Two special triangles can be used to find some values of sin θ, cos θ and tan θ. The values are given as fractions.





Example 1

Without using a calculator, find:

(a) sin 45° (b) cos 60° (c) tan 30°

Solution

(a) $\sin 45° = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{1}{\sqrt{2}}$

(b) $\cos 60° = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{1}{2}$

(c) $\tan 30° = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{1}{\sqrt{3}}$

Skill Exercises: Expressing Trigonometric Ratios as Fractions

1. Use the two special triangles on the previous page to fill in the missing values in this table. Give your answer as fractions.

θ	Sin θ	Cos θ	Tan θ
30º			$\frac{1}{\sqrt{3}}$
45º	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	
60º			

2. Use your calculator to fill in the missing values in the table. Give your answers as decimals.

θ	Sin θ	Cos θ	Tan θ
0º			
30º			
45º			
60º			
90º			

3. Use the tables in question 1 and 2 to change these fractions to decimals.

(a) $\frac{1}{2}$

(b) $\frac{{}^{-}1}{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{{}^{-}1}{\sqrt{2}}$

(e) $\frac{\sqrt{3}}{2}$

(f) $\frac{{}^{-}\sqrt{3}}{2}$

(g) $\frac{1}{\sqrt{3}}$

(h) $\frac{{}^{-}1}{\sqrt{3}}$

(i) $\frac{\sqrt{3}}{1}$

(j) $\frac{{}^{-}\sqrt{3}}{1}$

Skill Exercise: Drawing the Sine Curve

The table below gives the sine of some angles between 0° and 360°.

(a) Draw a set of axes like the one below.

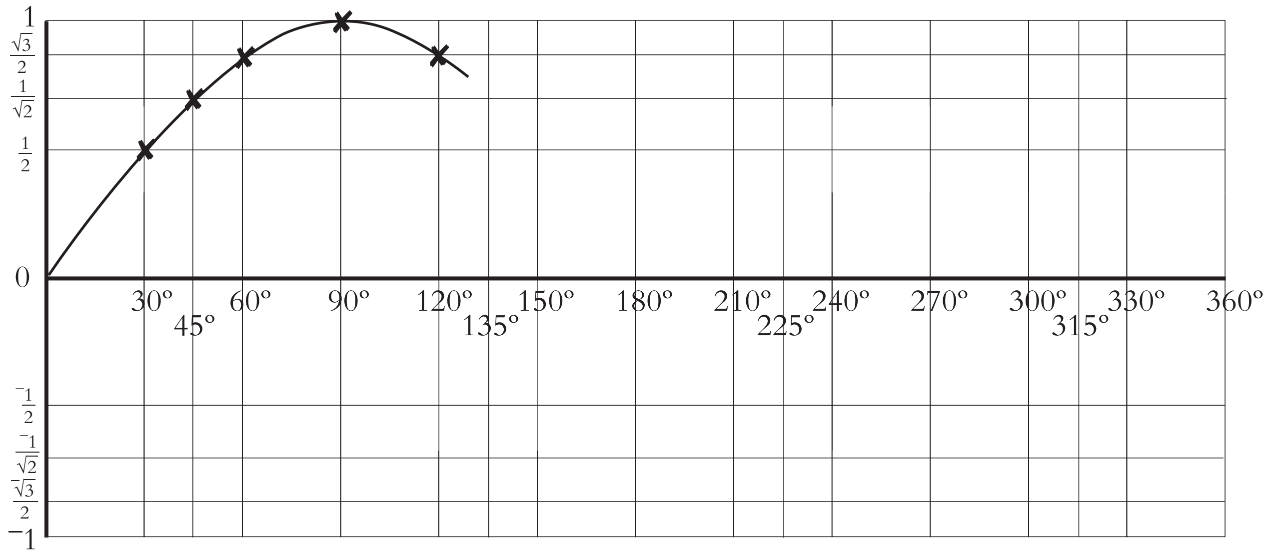
(b) Plot all the points.

(c) Join the points with a smooth curve.

The first part of the curve has been done.

Table of sin values

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{^-1}{2}$	$\frac{^-1}{\sqrt{2}}$	$\frac{^-\sqrt{3}}{2}$	$^-1$	$\frac{^-\sqrt{3}}{2}$	$\frac{^-1}{\sqrt{2}}$	$\frac{^-1}{2}$	0



Skill Exercise: Drawing the Cosine Curve

The table below gives the cosine of some angles between 0° and 360°

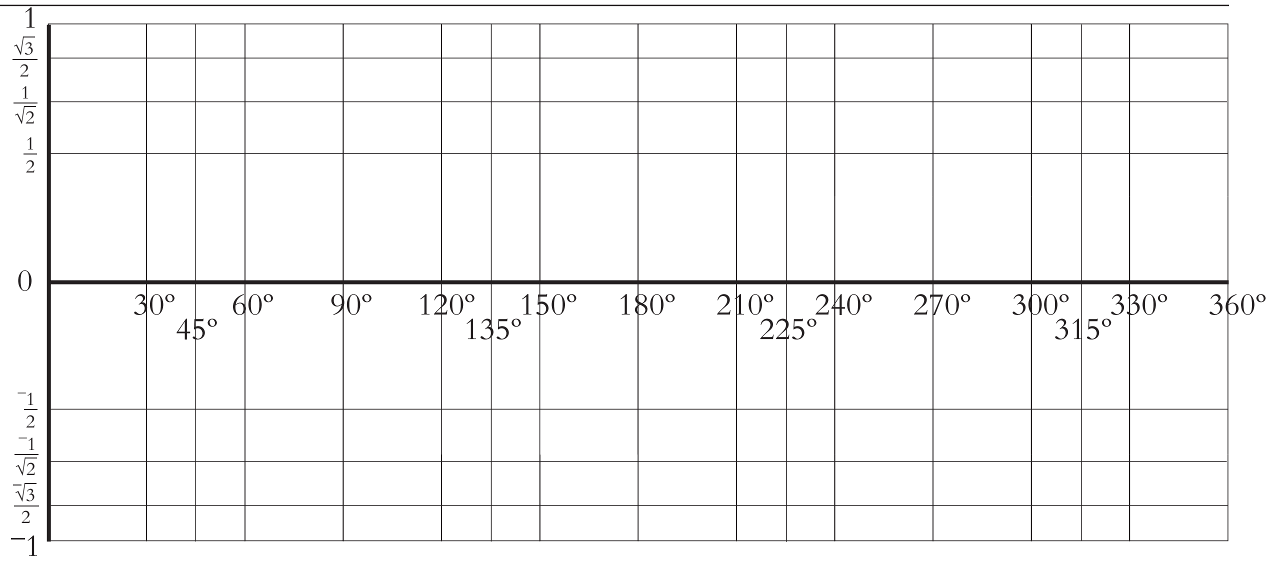
(a) Draw a set of axes like the one on the opposite page.

(b) Plot all the points.

(c) Join the points with a smooth curve.

Table of cosine values

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{^-1}{2}$	$\frac{^-1}{\sqrt{2}}$	$\frac{^-\sqrt{3}}{2}$	$^-1$	$\frac{^-\sqrt{3}}{2}$	$\frac{^-1}{\sqrt{2}}$	$\frac{^-1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1



Skill Exercise: Drawing the Tangent Curve

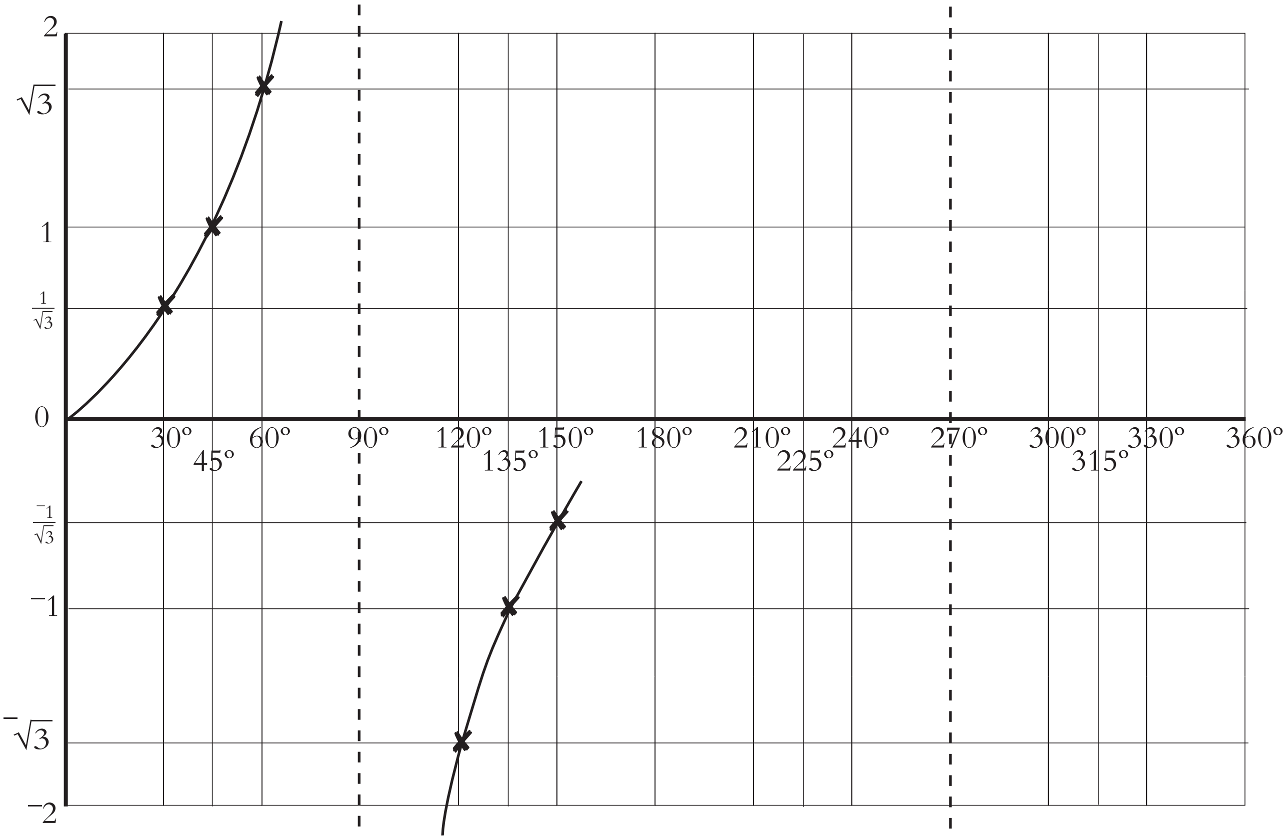
The table below gives the tangent of some angles between 0° and 360°:

(a) Draw a set of axes like the one below.

(b) Plot the points – Notice you cannot plot tan 90° or tan 270° because the values for tan 90° and tan 270° are infinity. The first few points have been plotted.

Table of tangent values

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$^-\sqrt{3}$	$^-1$	$\frac{^-1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$^-\sqrt{3}$	$^-1$	$\frac{^-1}{\sqrt{3}}$	0



Unit 6: GEOMETRY

In this unit you will be:

6.1 Using Bearings to Describe Direction and Plot Courses

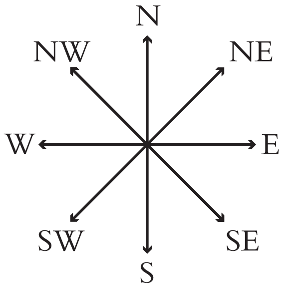
6.2 Constructing Scale Drawings

- ◧ Drawing Plans.
- ◧ Working with Enlargements.

6.3 Using the Properties of Circles to Solve Problems

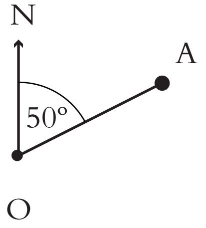
- ◧ Angles in Semi-circles.
- ◧ Using the Five Proofs.

Section 6.1 Using Bearings To Describe Direction And Plot Courses

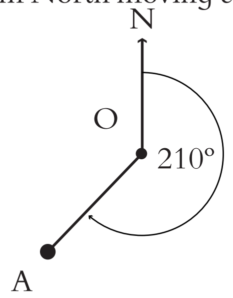


When describing a direction, the points of a compass are useful, e.g. S or SW.

A bearing is a direction measured in degrees from North moving clockwise.



The bearing of A from O is 050°.



The bearing of A from O is 210°.

A bearing is written using 3 figures.

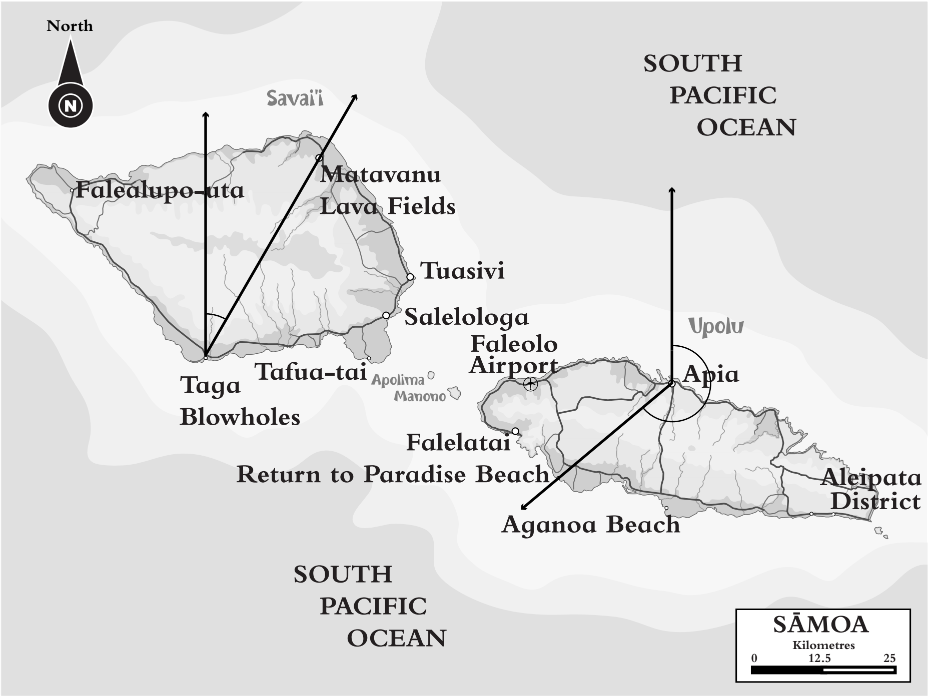
Example 1

On a map of Samoa find the bearings of:

(a) Matavanu from the Taga Blowholes.

(b) Return to Paradise Beach from Apia.

Solution



(a) Draw a line North from Taga Blowholes. Draw another line from Taga Blowholes to Matavanu. Measure the angle clockwise from North to the second line. In this example the angle is 30° so the bearing is 030°.

(b) Draw a line North from Apia. Draw another line from Apia to Return to Paradise Beach. Measure the angle clockwise from North to the second line. In this example the angle is 230° so the bearing is 230°.

Skill Exercises: Using Bearings to Describe Direction and Plot Courses

1. The diagram shows the positions of 8 children.

   (a) Who is directly south of Rachel?

   (b) If Mele walks SE, whom will she meet?

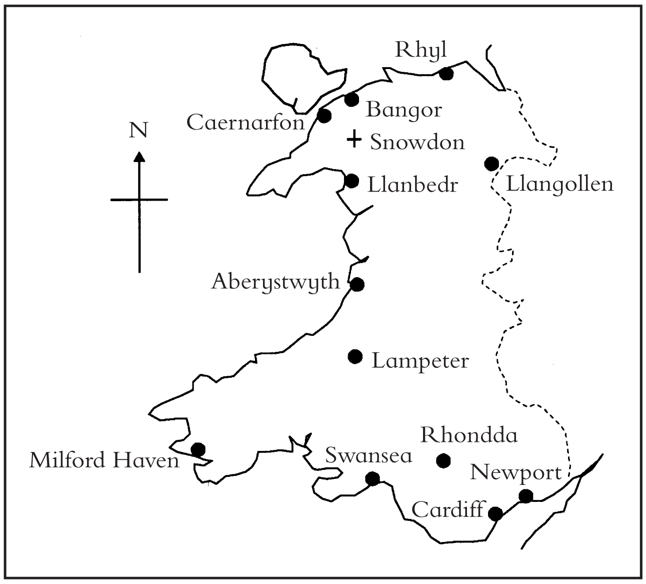
   (c) If Rachel walks SW, whom will she meet?

   (d) Who is directly west of Fereita?

   (e) Who is NW of Fereita?

   (f) Who will Simona meet if he walks NW?

   (g) In what direction should Eseta walk to find Rachel?



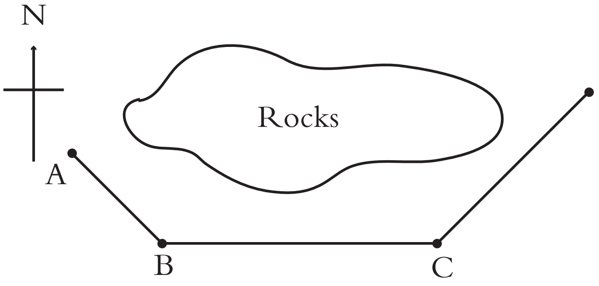
2. The map shows some towns and cities in Wales.

   Write down the bearing of each of these following places from Snowdon:

   (a) Llangollen (b) Newport (c) Swansea

   (d) Bangor (e) Milford Haven (f) Aberystwyth

3. In order to avoid an area of dangerous rocks a yacht sails as shown in the diagram.



   (a) Find the bearing of the yacht as it sails from:

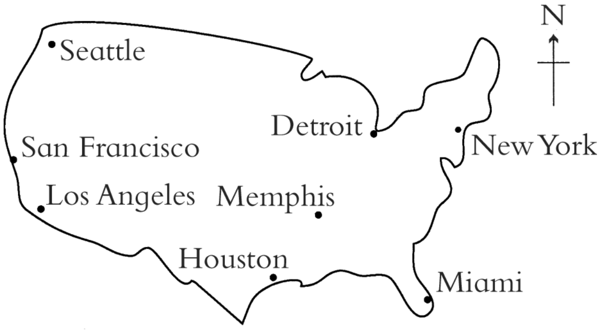
      (i) A to B (ii) B to C (iii) C to D

   (b) How much further does the yacht travel to avoid the rocks?

4. A rough map of the USA is shown below.

   Find the bearings of:

   (a) Miami from Memphis
   (b) Detroit from New York
   (c) Los Angeles from Detroit
   (d) Seattle from Houston
   (e) Memphis from San Francisco



5. Use a scale drawing to find the answers of each of the following problems.

   (a) If a man walks 700 m on a bearing of 040°, how far north and how far east is he from his starting point?

   (b) A dog runs 50 m on a bearing of 230° and then runs north until he is west of his starting point. How far is the dog from its starting point?

   (c) A helicopter flies 80 km north and then 20 km SW. What bearing would have taken the helicopter directly to its final position?

      How far is the helicopter from its starting point?

   (d) A boat travels 500 m NE and then 500 m south. What bearing would take the boat back to its starting point?

   (e) A plane flies 300 km west and then a further 200 km SW. It then returns directly to its starting point.

      On what bearing should it fly and how far does it have to travel?

Section 6.2 Constructing Scale Drawings

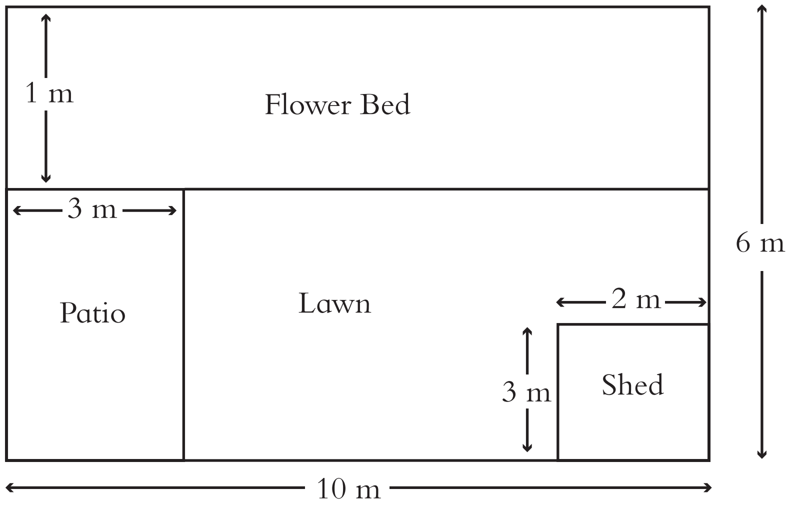
Drawing Plans

Scale drawings are often used to produce plans for houses or new kitchens.

For example, a scale of 1: 40 means that 1 cm on the plan /is equivalent to 40 cm in reality.

Example 1

The diagram shows a rough layout of a garden.



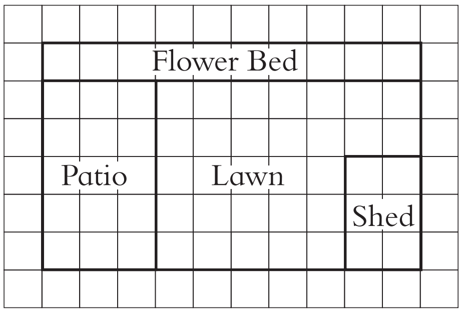
Produce a scale drawing with a scale of 1:200.

Solution

A scale of 1: 200 means that 1 cm on the plan represents 200 cm or 2 m in reality.The table below lists the sizes of each part of the garden and the size on the scale drawing.

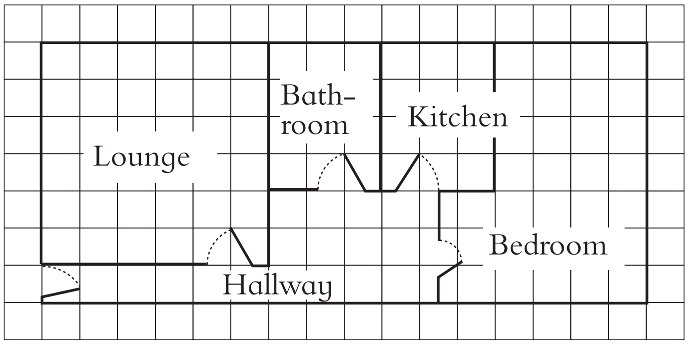
Area	Real Size	Size on Drawing
Garden	10 m × 6 m	5 cm × 3 cm
Patio	5 m × 3 m	2.5 cm × 1.5 cm
Lawn	7 m × 5 m	3.5 cm × 2.5 cm
Flower Bed	10 m × 1 m	5 cm × 0.5 cm
Shed	3 m × 2m	1.5 cm × 1 cm

A scale drawing now can be produced and is shown opposite drawn on squared paper.



Skill Exercises: Drawing Plans

1. The scale drawing below of a flat has been drawn on a scale of 1: 200.



   (a) Find the actual size of the lounge.

   (b) Find the area of the bedroom floor.

   (c) Find the length of the hallway, from the front door to the bedroom.

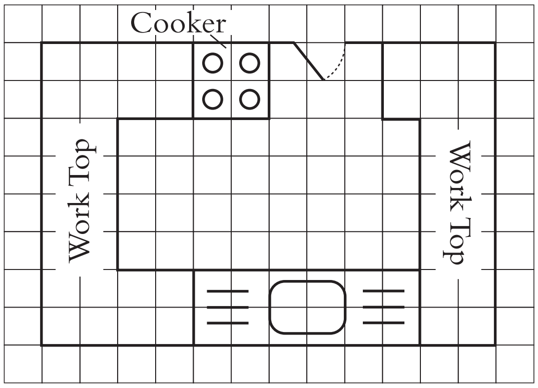
2. The scale drawing shows a plan of a kitchen on a scale of 1: 60.

   (a) What are the length and width of the kitchen?

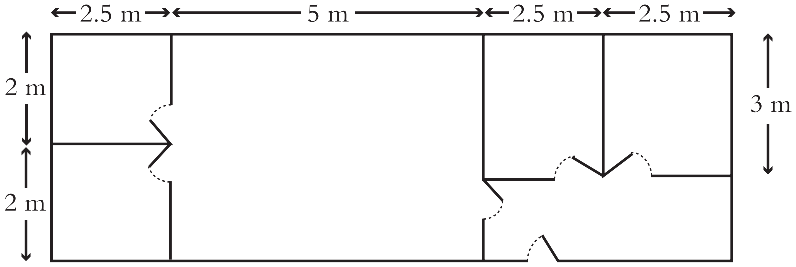
   (b) What is the size of the cooker?

   (c) What is the size of the sink?

   (d) Find the area of the worktops in the kitchen.



3. A rough sketch is made of a set of offices. It is shown below:



   Use the information given to produce a scale drawing with a scale of 1: 200.

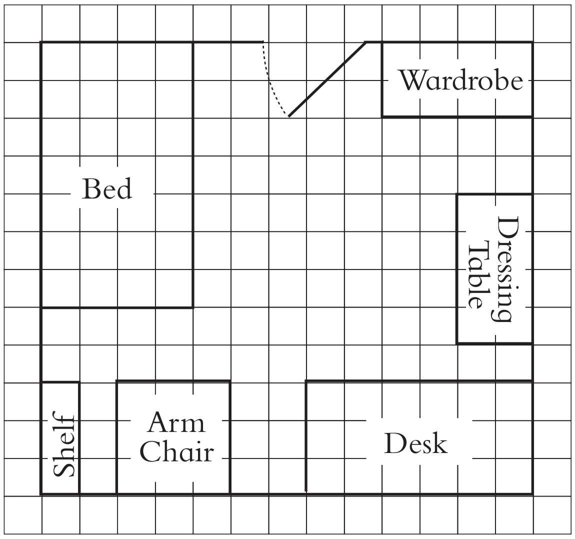
4. Hannah produces a scale drawing of her ideal bedroom on a scale of 1: 50. The plan is shown below:

   (a) What is the size of the room?

   (b) How long is her bed?

   (c) What is the area of the top of her desk?

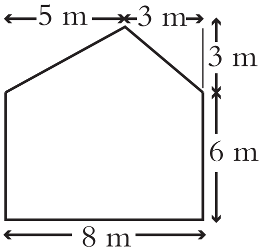
   (d) What is the floor area of the room?



5. The diagram shows a rough sketch of the end wall of the house.

   (a) Produce a scale drawing using a scale of 1: 100.

   (b) Use the drawing to find the sloping lengths of both sides of the roof.



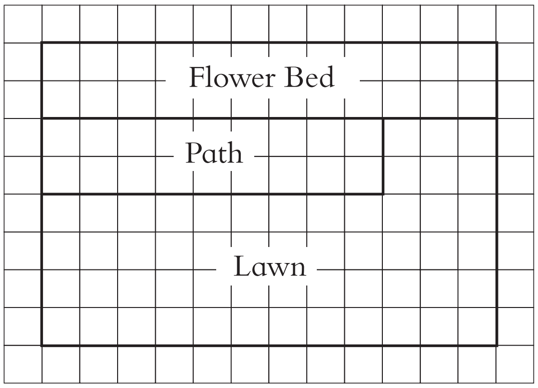
6. The diagram shows a scale drawing of the garden. It is drawn with a scale of 1:80.

   (a) How long is the garden?

   (b) How long is the path?

   (c) A shed with a base of size 2 m by 1.5 m is to be added to the plan. Find the size of the rectangle that should be drawn on the plan.

   (d) The garden also contains a pond of radius 0.6 m. What would be the radius of the circle which should be added to the plan?



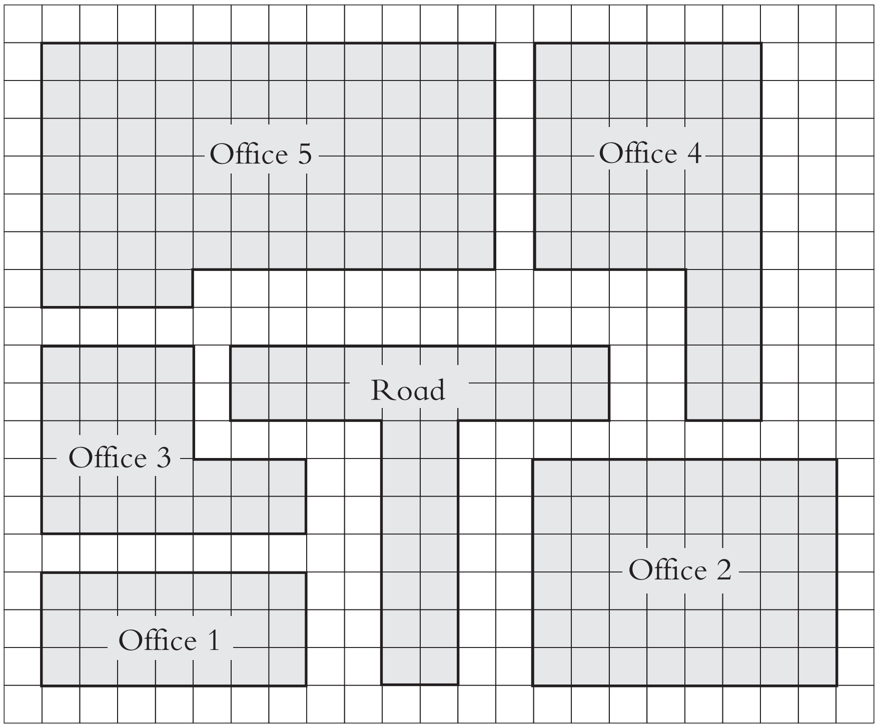
7. A classroom is rectangular with width 4 m and length 5 m. What would be the size of the rectangle used to represent the classroom on plans with a scale of:

   (a) 1: 50 (b) 1: 25 (c) 1: 100?

8. The diagram shows a scale plan of some office buildings drawn on a scale of 1:500.

   (a) Find the floor area of each office.

   (b) Re-draw the plan with a scale of 1:1000.



9. The plan of a house has been drawn using a scale of 1: 20.

   (a) On the plan, the length of the lounge is 25 cm. What is the actual length of the lounge in metres?

   (b) The actual lounge is 3.2 m wide. How wide is the lounge on the plan?

10. A classroom is drawn on a plan using a scale of 1: 50.

    (a) On the plan, how many centimetres represent one metre?

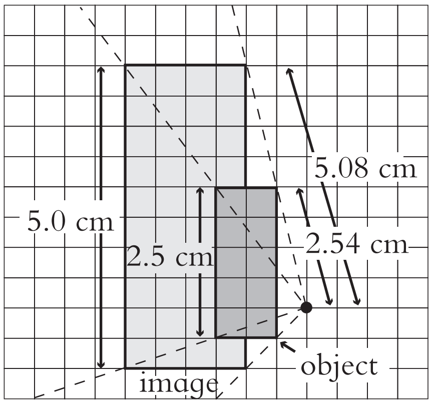
    (b) The width of the classroom is 6.7 m. How many centimetres represent this width on the plan?

Working With Enlargements

Enlargement

An enlargement requires both a centre and a scale factor. The entire plane expands (or shrinks) towards (or from) the centre of enlargement.

Example 1



All points on the plane expand or shrink from the centre.

$$\text{Scale factor} = \frac{\text{distance to image}}{\text{distance to object}}$$

$$= \frac{5.08 \text{ cm}}{2.54 \text{ cm}}$$

$$= 2$$

or

$$\text{Scale factor} = \frac{\text{part of the image}}{\text{corresponding part of object}}$$

$$\text{Scale factor} = \frac{5.0 \text{ cm}}{2.5 \text{ cm}}$$

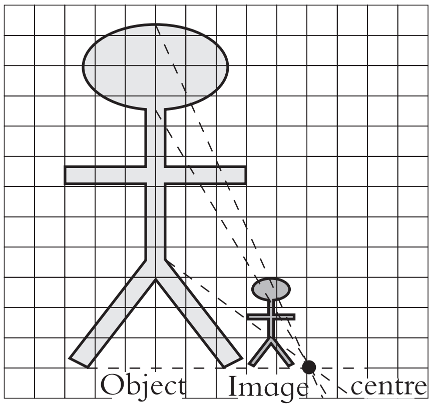
$$= 2$$

Centre of Enlargement

The centre of enlargement is found by joining corresponding parts of the object and the image.

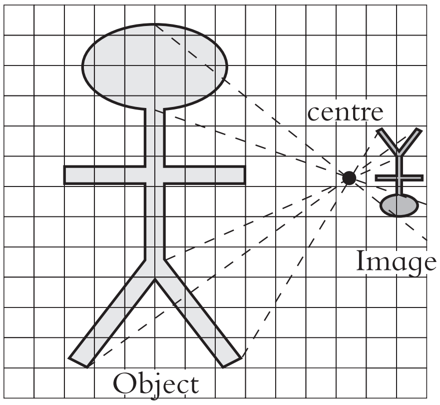
In Examples 2 and 3 the centre of enlargement is shown by joining corresponding points. In Example 3 the enlargement is negative (other side of the centre).

Example 2



A positive enlargement.

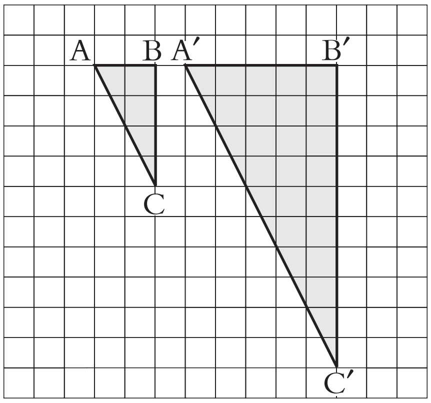
Example 3



A negative enlargement.

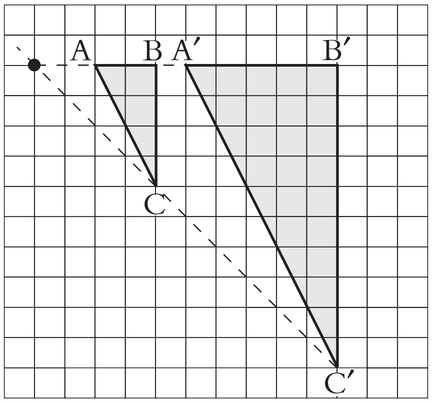
Example 4

For the enlargement below, find the centre and the scale factor. The image is A′B′C′.



Solution

Join corresponding points to find the centre.



The scale factor is found by the ratio of image to object.

$$\text{Scale factor} = \frac{\text{image}}{\text{object}}$$

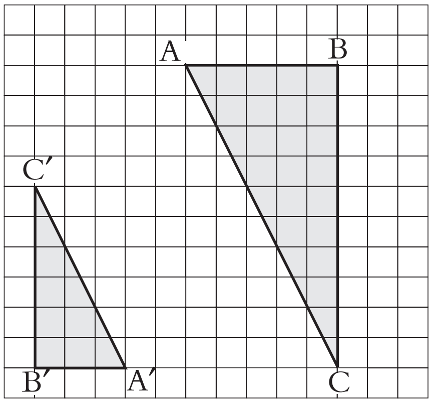
$$= \frac{A'B'}{AB}$$

$$= \frac{5}{2}$$

$$= 2.5$$

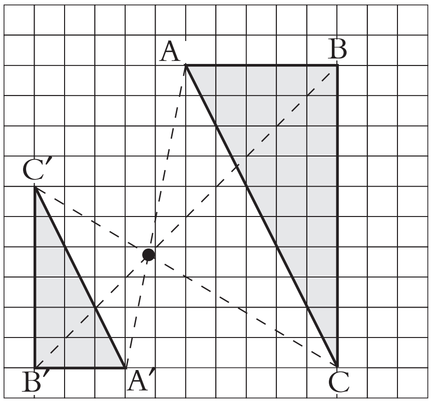
Example 5

For the enlargement below, find the centre and the scale factor. The image is A′B′C′.



Solution

Join corresponding points to find the centre.



The scale factor is found by the ratio of image to object. As the image is on the other side of the centre the enlargement is negative.

$$\text{Scale factor} = \frac{\text{image}}{\text{object}}$$

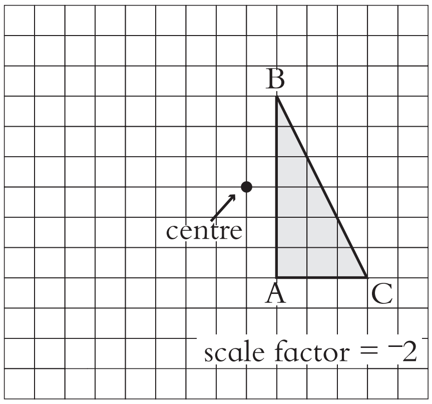
$$= \frac{A'B'}{AB}$$

$$= \frac{^{-}3}{5}$$

$$= {}^{-}0.6$$

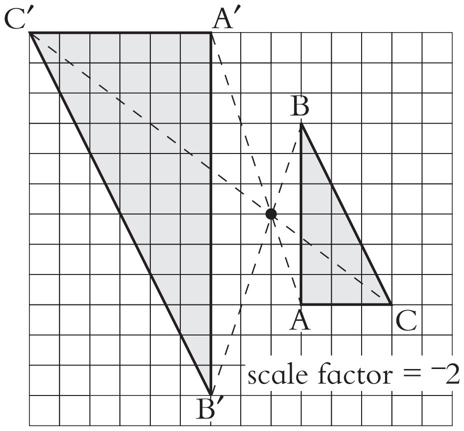
Example 6

Enlarge each object from the marked centre by the scale factor given.



Solution

Each vertex or corner is transformed so it is twice as far (scale factor 2) but on the other side of the centre (scale factor is negative).

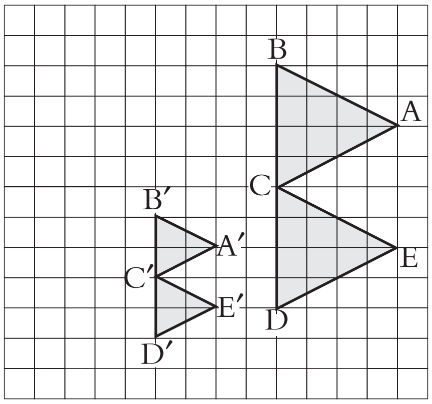


Any points are labelled. If the original is labelled A then the image is labelled A′.

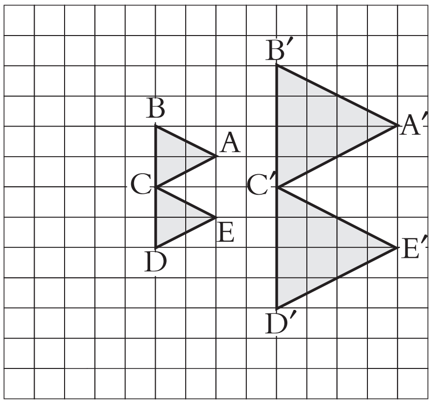
Skill Exercises: Working with Enlargements

For each enlargement find the centre and scale factor.

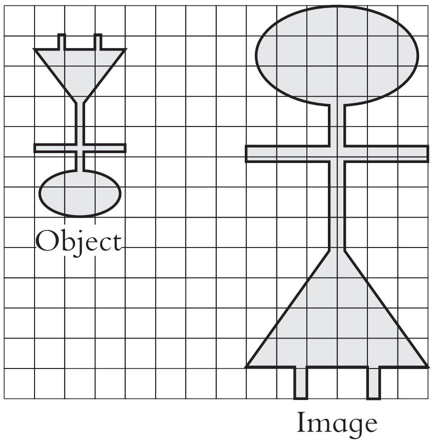
1. (a)



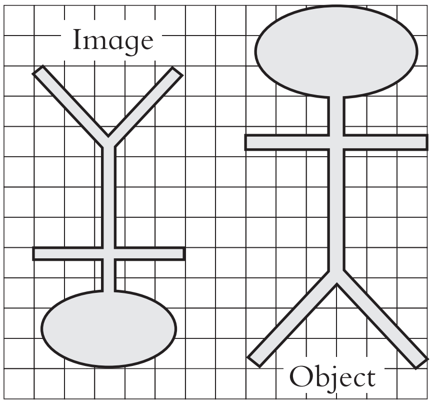
(b)



2. (a)

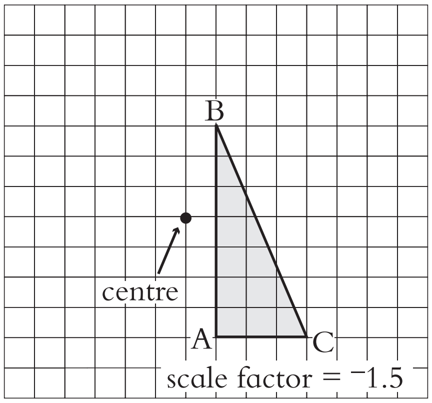


(b)

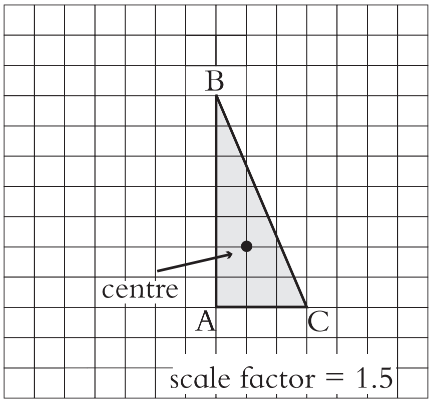


Enlarge each object from the marked centre by the scale factor given.

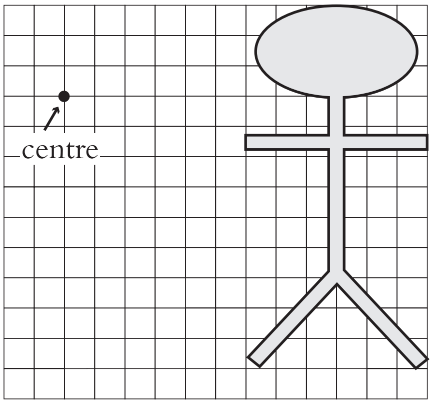
3. (a)



(b)

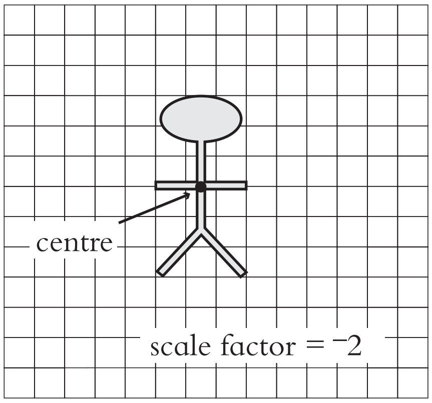


4. (a)



scale factor = $\frac{2}{3}$

(b)

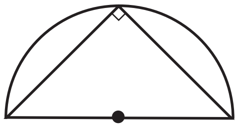


Section 6.3 Using The Properties Of Circles To Solve Problems

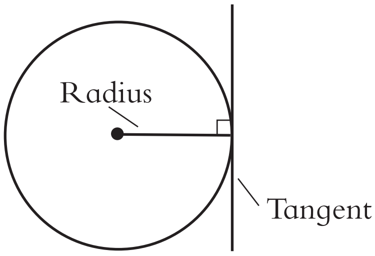
Angles in Semi-circles

The following results are true in any circle:

When a triangle is drawn in a semi-circle as shown; the angle on the circumference is always a right angle.



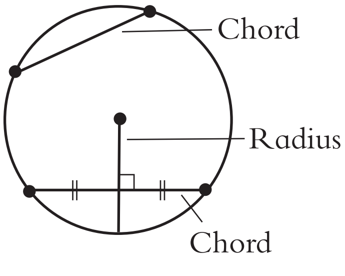
A *tangent* is a line that just touches a circle. It is always perpendicular to the radius.



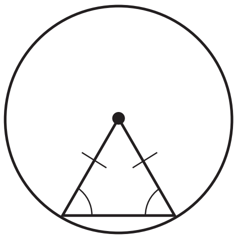
A *chord* is a line joining any two points on the circle.

The *perpendicular bisector* is a second line that cuts the first line in half and is at right angles to it.

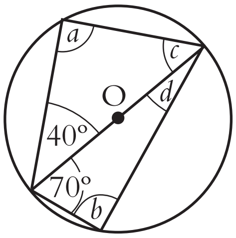
The perpendicular bisector of a chord is always a radius of the circle.



When the ends of a chord are joined to the centre of a circle, an isosceles triangle is formed, so the two base angles marked are equal.



Example 1



Find the angle marked with letters in the diagram, if O is the centre of the circle.

Solution

As both triangles are in semi-circles, angles *a* and *b* must each be 90°.

The other angles can be found because the sum of the angles in each triangle is 180°.

For the top triangle,

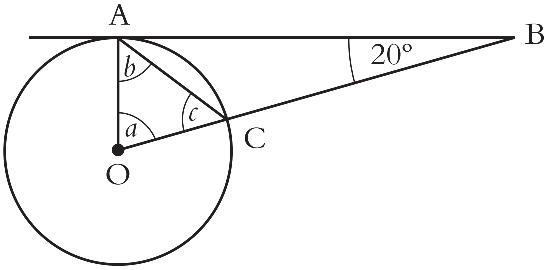
$$\begin{aligned} 40° + 90° + c &= 180° \\ c &= 180° - 130° \\ &= 50°. \end{aligned}$$

For the bottom triangle,

$$\begin{aligned} 70° + 90° + d &= 180° \\ d &= 180° - 160° \\ &= 20°. \end{aligned}$$

Example 2

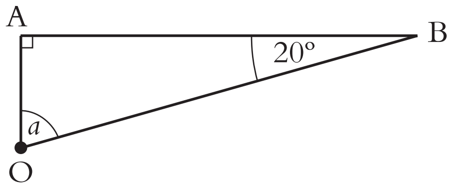
Find the angles *a*, *b* and *c*, if AB is a tangent and O is the centre of the circle.



Solution

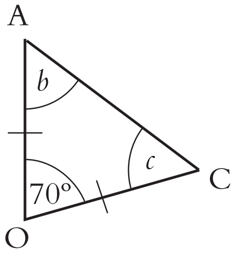
First consider the triangle OAB. As OA is a radius and AB is a tangent, the angle between them is 90°. So

$$\begin{aligned} 90° + 20° + a &= 180° \\ a &= 180° - 110° \\ &= 70°. \end{aligned}$$



Then consider the triangle OAC. As OA and OC are both radii of the circle, it is an isosceles triangle with $b = c$.

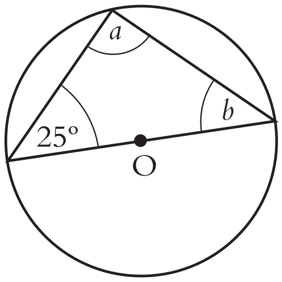
$$\begin{aligned} \text{So} \qquad 2b + 70° &= 180° \\ 2b &= 110° \\ b &= 55° \\ \text{and} \qquad c &= 55°. \end{aligned}$$



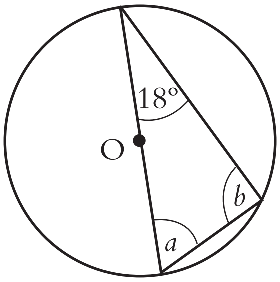
Skill Exercises: Angles in Semi-circles

1. Find the angles marked with a letter in each of the following diagrams. In each case the centre of the circle is marked O.

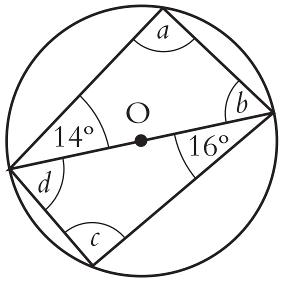
(a)



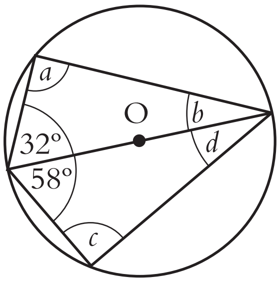
(b)



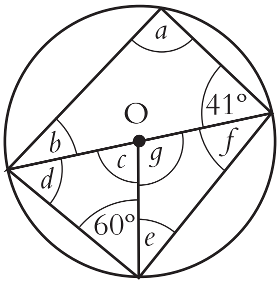
(c)



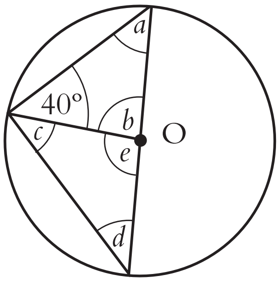
(d)



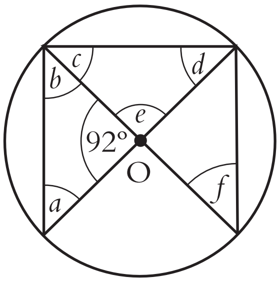
(e)



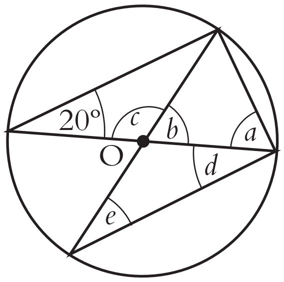
(f)



(g)

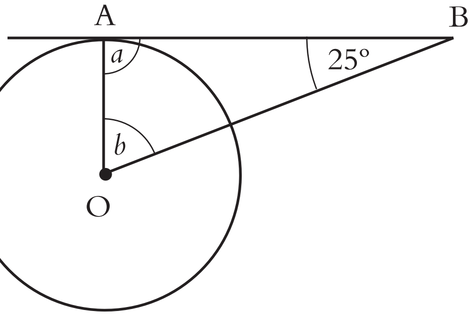


(h)

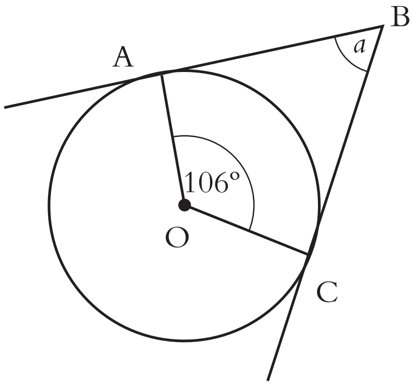


2. Find the angles marked with letters in each diagram below, if O is the centre of the circle.

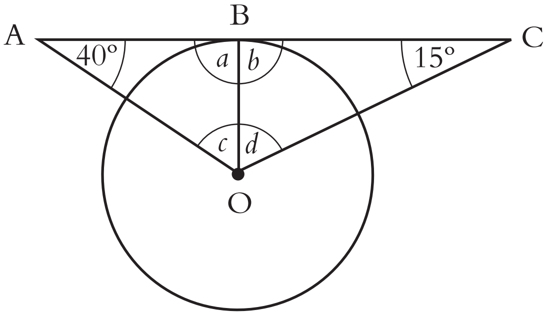
(a)



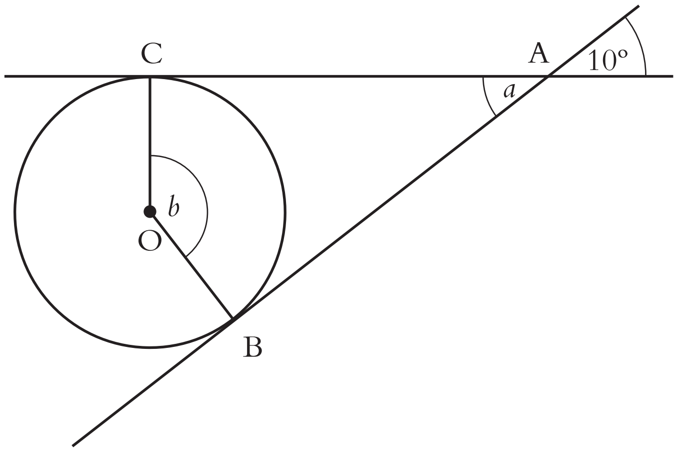
(b)



(c)

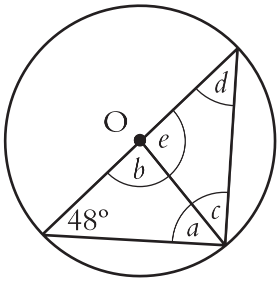


(d)

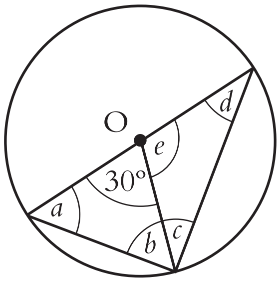


3. Find the angles marked with letters in each of the following diagrams, if O is the centre of the circle.

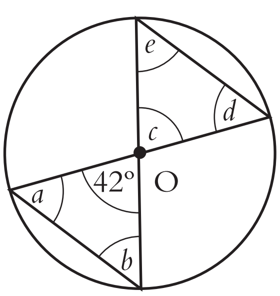
(a)



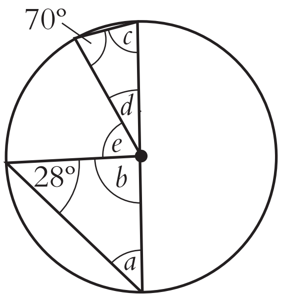
(b)



(c)

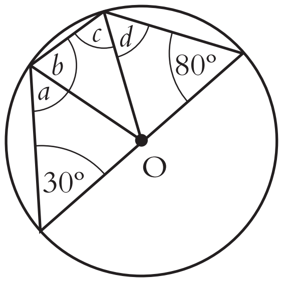


(d)

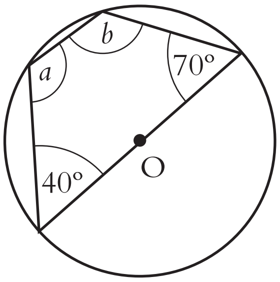


4. Find each of the marked angles if O is the centre of the circle.

(a)



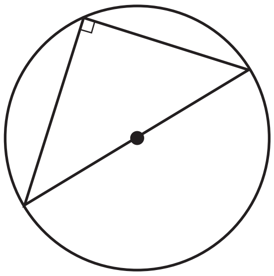
(b)



Using the Five Proofs

There are a number of important geometric results based on angles in circles. (The first you have met already.)

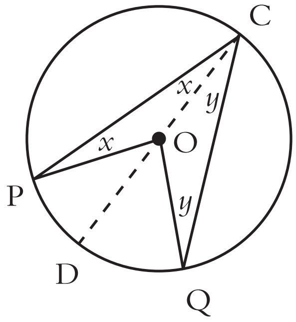
Proof 1



Any angle subtended on the circumference from a diameter is a right angle.

Proof 2

The angle subtended by an arc, PQ, at the centre is twice the angle subtended on the circumference.



OP = OC (equal radii), so

angle CPO = angle PCO (= x, say).

Similarly,

angle CQO = angle QCO (= y, say).

Now, extending the line CO to D, say, note that

$$\text{angle POD} = x + x = 2x$$

and, similarly,

$$\text{angle QOD} = y + y = 2y.$$

Hence,

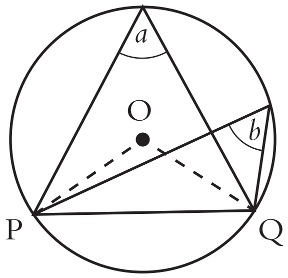
$$\begin{aligned}\text{angle POQ} &= 2x + 2y \\ &= 2(x + y) \\ &= 2 \times \text{angle PCQ as required.}\end{aligned}$$

Proof 3

Angles subtended at the circumference by a chord (on the same side of the chord) are equal; that is, in the diagram $a = b$.

The angle at the centre is $2a$ or $2b$ (according to the final result).

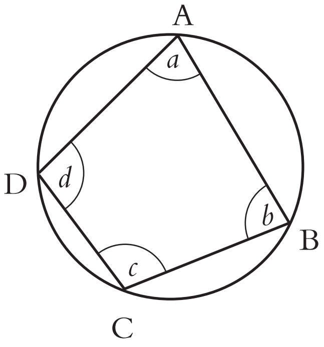
Thus $2a = 2b$ or $a = b$, as required.



Proof 4

In *cyclic quadrilaterals* (quadrilaterals where all 4 vertices lie on a circle), opposite angles sum to 180°;

$$\text{That is} \quad a + c = 180°$$
$$\text{And} \quad b + d = 180°.$$



Construct the diagonals AC and BD, as below.

Label the angles subtended by AB as w.

Angle ADB = angle ACB (= w).

For the other chords, the angles are marked x, y and z as shown.

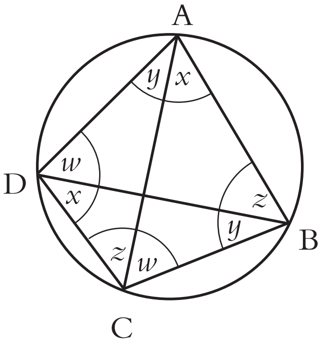
In triangle ABD, the sum of the angles is 180°,

$$\therefore \quad w + z + (x + y) = 180°.$$

This can be rearranged as

$$(x + w) + (y + z) = 180°,$$

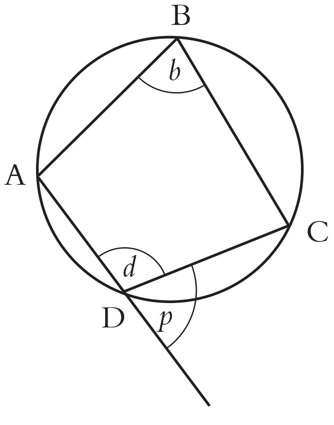
$\therefore$ angle CDA + angle CBA = 180°.



Proof 5

Exterior angles of cyclic quadrilaterals are equal in size to the interior and opposite angles.

$$\begin{aligned} p + d &= 180° \\ b + d &= 180° \quad \text{(see proof 4)} \\ \therefore p + d &= b + d \\ p &= b \\ &= 40°. \end{aligned}$$

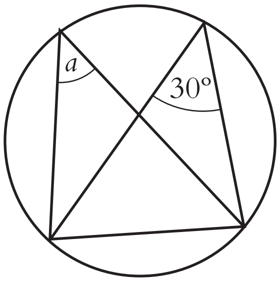


Skill Exercises: Using the Five Proofs

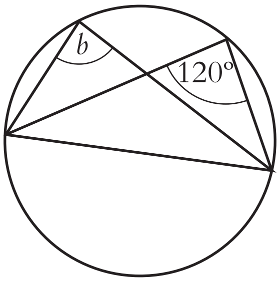
1. Find all the angles marked with a letter in each of the following diagrams.

   In each case the centre of the circle is marked O.

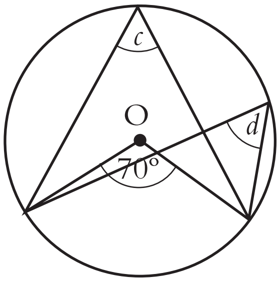
(a)



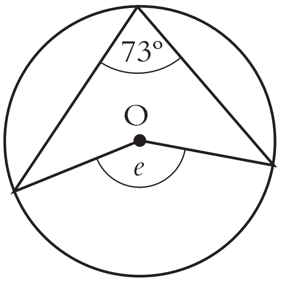
(b)



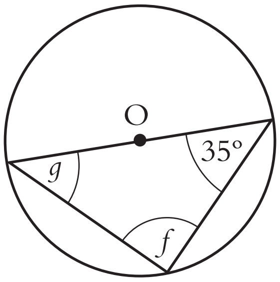
(c)



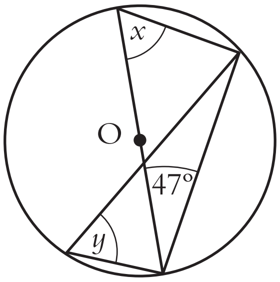
(d)



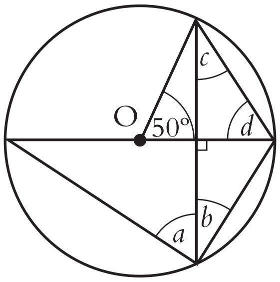
(e)



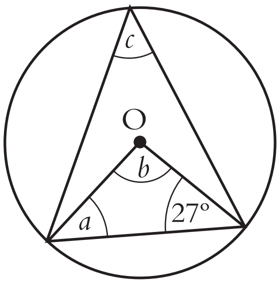
(f)



(g)

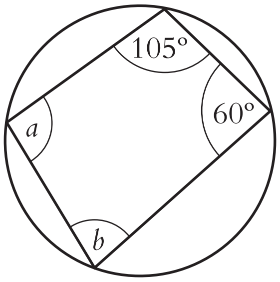


(h)

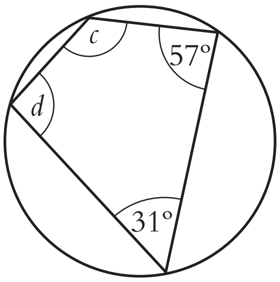


2. Find all of the angles marked with a letter in each of the following diagrams.

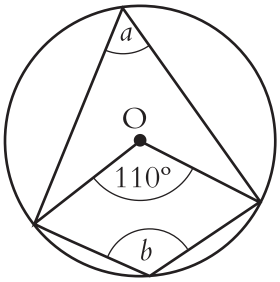
(a)



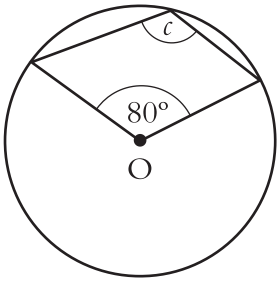
(b)



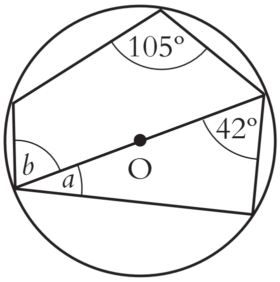
(c)



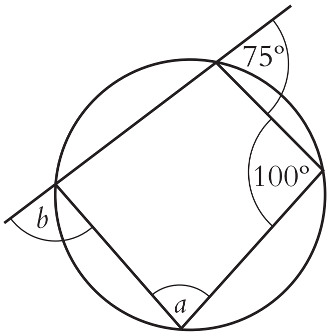
(d)



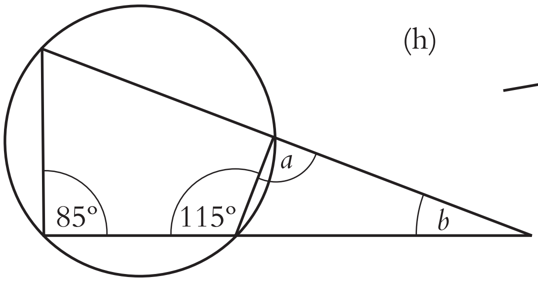
(e)



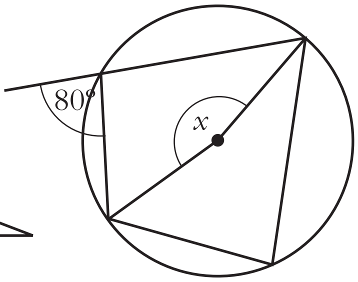
(f)



(g)

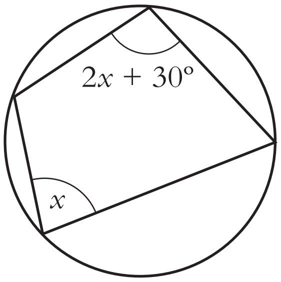


(h)

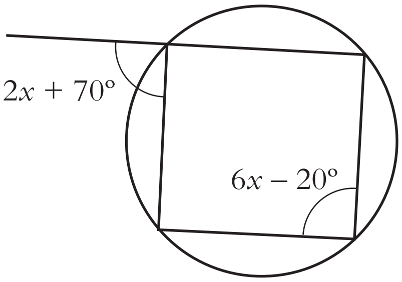


3. Find the value of *x* in each of the following diagrams.

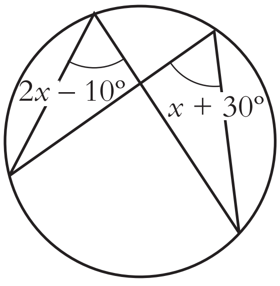
(a)



(b)



(c)



Unit 7: STATISTICS

In this unit you will be:

7.1 Distinguishing Different Types of Data

- ◧ Discrete and Continuous Data.
- ◧ Grouped and Ungrouped Data.

7.2 Working with Data

- ◧ Mean, Median, Mode and Range.
- ◧ Finding the Mean from Tables and Tally Charts.
- ◧ Calculations with the Mean.
- ◧ Mean, Median and Mode for Grouped Data.
- ◧ Cumulative Frequency.

Section 7.1 Distinguishing Different Types Of Data

Discrete and Continuous Data

Raw data is the information collected in a survey. This data can either be discrete or continuous.

Discrete data is collected by counting. The data is in the form of whole numbers.

e.g. The number of eggs in a bird's nest = 4

The number of grains of rice in a bag = 10 956

Continuous data is collected by measuring. It can be in the form of decimals.

e.g. The weight of a bird's egg = 52.36 grams

The length of a grain of rice = 0.85 cm

Skill Exercises: Discrete and Continuous Data

Read the sentences below. Decide whether the data is discrete or continuous.

(a) The heights of students in class 11 A.

(b) The number of biscuits in packets of five different brands.

(c) The times taken for your class to run 100 metres.

(d) The lengths of fish caught in a net.

(e) The number of videos rented out each week.

(f) The weights of the players in Manu Samoa.

(g) The number of goals scored in a season by netball teams.

Grouped and Ungrouped Data

After data has been collected you may have a long list of numbers. These numbers can be put into groups to make it easier to understand them. The information can be shown on a frequency table.

Example

The ages of a group of teachers at a meeting were collected. The ungrouped (raw) data is:

21	28	40	62	41	38	35	25	32	32
40	29	38	35	48	47	57	67	23	21
48	43	30	53	32	37	27	41		

Draw a frequency table in which this data is grouped in 10 year intervals.

Solution

The Grouped Data is:

Age Group	Tally	Frequency
20–29	𝍸 II	7
30–39	𝍸 IIII	9
40–49	𝍸 III	8
50–59	II	2
60–69	II	2
Total		28

Skill Exercises: Grouped and Ungrouped Data

1. Thirty students were given a maths test. Their scores are shown below. Draw a frequency table in which this data is grouped in intervals of 10.

62	61	58	54	55	40
42	27	78	75	56	64
48	64	73	94	49	48
52	51	55	82	84	27
34	58	66	64	72	37

2. Twenty Year 11 students had their heights measured. The ungrouped data is shown below. Draw a frequency table in which this data is grouped in intervals of 5 cm.

1.68 m	1.72 m	1.56 m	1.69 m	1.62 m
1.78 m	1.66 m	1.64 m	1.73 m	1.68 m
1.76 m	1.61 m	1.61 m	1.56 m	1.69 m
1.71 m	1.66 m	1.64 m	1.66 m	1.71 m

Section 7.2 Working With Data

Mean, Median, Mode and Range

In this unit, you will find out how to calculate statistical quantities which summarise the important characteristics of the data.

The *mean*, *median* and *mode* are three different ways of describing the average.

- To find the *mean*, add up all the numbers and divide by the number of numbers.
- To find the *median*, place all the numbers in order and select the middle number.

- The *mode* is the number which appears most often.
- The *range* gives an idea of how the data are spread out and is the difference between the smallest and largest values.

Example 1

Find

(a) the mean (b) the median (c) the mode (d) the range

of this set of data:

5, 6, 2, 4, 7, 8, 3, 5, 6, 6

Solution

(a) The mean is

$$\frac{5+6+2+4+7+8+3+5+6+6}{10}$$

$$= \frac{52}{10}$$

$$= 5.2$$

(b) To find the median, place all the numbers in order.

2, 3, 4, 5, 5, 6, 6, 6, 7, 8

As there are *two* middle numbers in this example, 5 and 6,

$$\text{median} = \frac{5+6}{2}$$

$$= \frac{11}{2}$$

$$= 5.5$$

(c) From the list on the previous page it is easy to see that 6 appears more than any other number, so

mode = 6.

(d) The range is the difference between the smallest and largest numbers, in this case 2 and 8. So the range is $8 - 2 = 6$.

Skill Exercises: Mean, Median, Mode and Range

1. Find the mean, median, mode and range of each set of numbers below:
    (a) 3, 4, 7, 3, 5, 2, 6, 10
    (b) 8, 10, 12, 14, 7, 16, 5, 7, 9, 11
    (c) 17, 18, 16, 17, 17, 14, 22, 15, 16, 17, 14, 12
    (d) 108, 99, 112, 111, 108
    (e) 64, 66, 65, 61, 67, 61, 57
    (f) 21, 30, 22, 16, 24, 28, 16, 17

2. Twenty students were asked their shoe sizes. The results are shown below:

8,	6,	7,	6,	5.	$4\frac{1}{2}$,	$7\frac{1}{2}$,	$6\frac{1}{2}$,	$8\frac{1}{2}$,	10
7,	5,	$5\frac{1}{2}$,	8,	9,	7,	5,	6,	$8\frac{1}{2}$,	6

For this data, find:

(a) the mean (b) the median (c) the mode (d) the range.

3. Eight people work in a shop. They are paid hourly rates of:

$2, $15, $5, $3, $4, $3, $3 $4.

(a) Find:
    (i) the mean (ii) the median (iii) the mode.

(b) Which average would you use if you wanted to claim that the staff were:
    (i) well paid (ii) badly paid?

(c) What is the range?

4. Two people work in a factory making parts for cars. The table shows how many complete parts they make in one week.

Worker	Mon	Tue	Wed	Thu	Fri
Fred	21	21	22	20	20
Henry	30	15	12	36	28

(a) Find the mean and range for Fred and Harry.

(b) Who is the most consistent?

(c) Who makes the most parts in a week?

5. A gardener buys 10 packets of seeds from two different companies. Each pack contains 20 seeds and he records the number of plants which grow from each pack.

Company A	20	5	20	20	20	6	20	20	20	8
Company B	17	18	15	16	18	18	17	15	17	18

(a) Find the mean, median and mode for each company's seeds.

(b) Which company does the mode suggest is best?

(c) Which company does the mean suggest is best?

(d) Find the range for each company's seeds.

6. Aukuso takes four tests and scores the following marks.

65, 72, 58, 77

(a) What are his median and mean scores?

(b) If he scores 70 in his next test, does his mean score increase or decrease?

(c) Which has increased most, his mean score or his median score?

7. Felise keeps a record of the number of fish he catches over a number of fishing trips. His records are:

1, 0, 2, 0, 0, 0, 12, 0, 2, 1, 18, 0, 2, 0, 1.

(a) Why does he not like to talk about the mode and median of the number of fish caught?

(b) What are the mean and range of the data?

(c) Felise's friend, Talia, also goes fishing. The mode of the number of fish she has caught is also 0 and the range is 15. What is the largest number of fish Talia has caught?

8. A petrol station owner records the number of cars which visit his garage on 10 days. The numbers are:

204, 310, 279, 314, 257, 302, 232, 261, 308, 217.

(a) Find the mean number of cars per day.

(b) The owner hopes that the mean will increase if he includes the number of cars on the next day. If 252 cars use the garage on the next day, will the mean increase or decrease?

9. The students in a class say how many children there are in their family. The numbers are given below.

1, 2, 1, 3, 2, 1, 2, 4, 2, 2, 1, 3, 1, 2,
2, 2, 1, 1, 7, 3, 1, 2, 1, 2, 2, 1, 2, 3

(a) Find the mean, median and mode for this data.

(b) Which is the most sensible average in this case?

10. The mean number of people visiting Julia each day over a five-day period is 8. If 10 people visit Julia the next day, what happens to the mean?

11. The table shows the maximum and minimum temperatures recorded in six cities one day last year.

City	Maximum	Minimum
Los Angeles	22°C	12°C
Boston	22°C	⁻3°C
Moscow	18°C	⁻9°C
Atlanta	27°C	8°C
Archangel	13°C	⁻15°C
Cairo	28°C	13°C

(a) Work out the range of temperatures for Atlanta.

(b) Which city in the table had the lowest temperature?

(c) Work out the difference between the maximum temperature and the minimum temperature for Moscow.

12. The weights, in grams, of seven talo are

260, 225, 205, 240, 232, 205, 214.

What is their median weight?

Finding the Mean from Tables and Tally Charts

Often data are collected into tables or tally charts. This section considers how to find the mean in such cases.

Example 1

A soccer team keeps records of the number of goals it scores per match during a season.

No. of Goals	Frequency
0	8
1	10
2	12
3	3
4	5
5	2

Find the mean number of goals per match.

Solution

The table above can be used, with a third column added.

No. of Goals	Frequency	No. of Goals × Frequency
0	8	$0 \times 8 = 0$
1	10	$1 \times 10 = 10$
2	12	$2 \times 12 = 24$
3	3	$3 \times 3 = 9$
4	5	$4 \times 5 = 20$
5	2	$5 \times 2 = 10$
TOTALS	40	73
	(Total matches)	(Total goals)

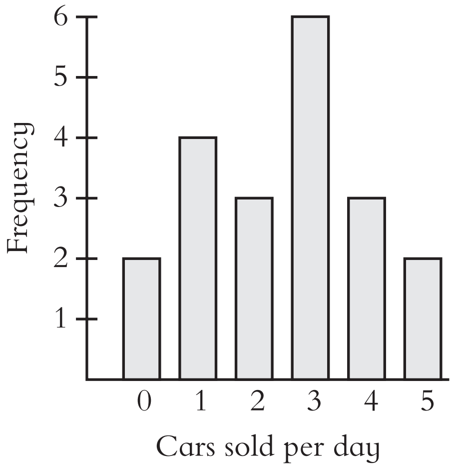
The mean can now be calculated.

$$\text{Mean} = \frac{73}{40}$$

$$= 1.825$$

Example 2

The bar chart shows how many cars were sold by a salesman over a period of time.



Find the mean number of cars sold per day.

Solution

The data can be transferred to a table and a third column included as shown:

Cars sold daily	Frequency	Cars sold × Frequency
0	2	$0 \times 2 = 0$
1	4	$1 \times 4 = 4$
2	3	$2 \times 3 = 6$
3	6	$3 \times 6 = 18$
4	3	$4 \times 3 = 12$
5	2	$5 \times 2 = 10$
TOTALS	20	50

(Total days) (Total number of cars sold)

$$\text{Mean} = \frac{50}{20}$$

$$= 2.5$$

Example 3

A police station kept records of the number of burglaries in their area each week for 100 days.The figures on the next page give the number of burglaries per week.

1	4	3	5	5	2	5	4	3	2	0	3	1	2	2	3	0	5	2	1
3	3	2	6	2	1	6	1	2	2	3	2	2	2	2	5	4	4	2	3
3	1	4	1	7	3	3	0	2	5	4	3	3	4	3	4	5	3	5	2
4	4	6	5	2	4	5	5	3	2	0	3	3	4	5	2	3	3	4	4
1	3	5	1	1	2	2	5	6	6	4	6	5	8	2	5	3	3	5	4

Find the mean number of burglaries per week.

Solution

The first step is to draw out and complete a tally chart. The final column shown below can then be added and completed.

No. of burglaries	Tally	Frequency	No. of burglaries × Frequency
0	||||	4	$0 \times 4 = 4$
1	𝍸 𝍸	10	$1 \times 10 = 10$
2	𝍸 𝍸 𝍸 𝍸 ||	22	$2 \times 22 = 44$
3	𝍸 𝍸 𝍸 𝍸 |||	23	$3 \times 23 = 69$
4	𝍸 𝍸 𝍸 |	16	$4 \times 16 = 64$
5	𝍸 𝍸 𝍸 ||	17	$5 \times 17 = 85$
6	𝍸 |	6	$6 \times 6 = 36$
7	|	1	$7 \times 1 = 7$
8	|	1	$8 \times 1 = 8$
	TOTALS	100	323

$$\text{Mean number of burglaries per week} = \frac{323}{100}$$

$$= 3.23$$

Skill Exercises: Finding the Mean from Tables and Tally Charts

1. A survey of 100 homes asked how many TV sets there were in each household. The results are given below.

No. of TV sets	0	1	2	3	4
Frequency	5	70	21	3	1

Calculate the mean number of TV sets per household.

2. The survey of question 1 also asked how many video recorders there were in each household. The results are given below.

No. of video recorders	0	1	2	3	4	5
Frequency	2	30	52	8	5	3

Calculate the mean number of video recorders per household.

3. A manager keeps a record of the number of calls she makes each day on her mobile phone.

Number of calls per day	0	1	2	3	4	5	6	7	8
Frequency	3	4	7	8	12	10	14	3	1

Calculate the mean number of calls per day.

4. A cricket team keeps a record of the number of runs scored in each over.

No. of Runs	0	1	2	3	4	5	6	7	8
Frequency	3	2	1	6	5	4	2	1	1

Calculate the mean number of runs per over.

5. A class conducts an experiment in biology. They place a number of 1 m by 1 m square grids on the playing field and count the number of worms which appear when they pour water on the ground. The results obtained are given below:

6	3	2	1	3	2	1	3	0	1	0	3	2	1
1	4	0	1	2	0	1	1	2	2	2	4	3	1
1	1	2	3	3	1	2	2	2	1	7	1		

(a) Calculate the mean number of worms.

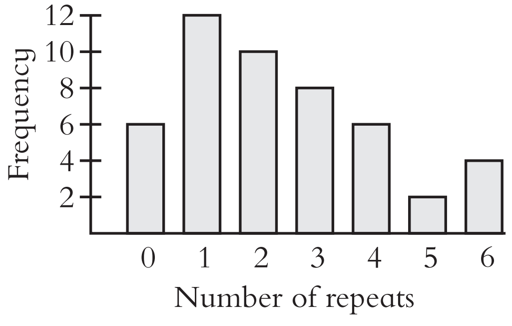
(b) How many times was the number of worms seen greater than the mean?

6. As part of a survey, a bus company recorded the number of buses which were late each day. The results are listed below:

0	1	2	4	1	0	2	1	1	0	1	2	1	3
1	0	0	0	0	5	2	1	3	2	0	1	0	1
2	1	1	0	0	3	0	1	2	1	0	0		

Construct a table and calculate the mean number of buses which were late each day.

7. Ana drew this bar chart to show the number of repeated cards she got when she opened packets of cereal.



Calculate the mean number of repeats per packet.

8. In a season a soccer team scored a total of 55 goals. The table below gives a summary of the number of goals per match.

Goals per Match	0	1	2	3	4	5
Frequency	4	6		8	2	1

   (a) In how many matches did they score 2 goals?

   (b) Calculate the mean number of goals per match.

9. A traffic warden is trying to work out the mean number of parking tickets he has issued per day. He produced the table below, but has accidentally rubbed out some of the numbers.

Tickets per day	Frequency	No. of Tickets × Frequency
0	1	
1		1
2	10	
3	7	
4		20
5	2	
6		
TOTALS	26	72

   Fill in the missing numbers and calculate the mean.

10. Here are the weights, in kg, of 30 students:

   45, 52, 56, 65, 34, 45, 67, 65, 34, 45, 65, 87, 45, 34, 56,

   54, 45, 67, 84, 45, 67, 45, 56, 76, 57, 84, 35, 64, 58, 60

   (a) Copy and complete the frequency table below using a class interval of 10 and starting at 30.

Weight Range (w)	Tally	Frequency
$30 \leq w < 40$		

   (b) Which class interval has the highest frequency?

11. The number of children per family in a recent survey of 21 families is shown.

1	2	3	2	2	4	2	2	3	2	2
2	3	2	2	2	4	1	2	3	2	

(a) What is the range in the number of children per family?

(b) Calculate the mean number of children per family. *Show your working.*

A similar survey was taken in 1960.

In 1960 the range in the number of children per family was 7 and the mean was 2.7

(c) Describe two changes that have occurred in the number of children per family since 1960.

Calculations with the Mean

This section considers calculations concerned with the mean, which is usually taken to be the most important measure of the average of a set of data.

Example 1

The mean of a sample of 6 numbers is 3.2. An extra value of 3.9 is included in the sample. What is the new mean?

Solution

$$\begin{aligned}\text{Total of original numbers} &= 6 \times 3.2 \\ &= 19.2 \\ \text{New total} &= 19.2 + 3.9 \\ &= 23.1 \\ \text{New mean} &= \frac{23.1}{7} \\ &= 3.3\end{aligned}$$

Example 2

The mean number of a set of 5 numbers is 12.7. What extra number must be added to bring the mean up to 13.1?

Solution

$$\begin{aligned}\text{Total of the original numbers} &= 5 \times 12.7 \\ &= 63.5 \\ \text{Total of the new numbers} &= 6 \times 13.1 \\ &= 78.6 \\ \text{Difference} &= 78.6 - 63.5 \\ &= 15.1\end{aligned}$$

So the extra number is 15.1.

Skill Exercises: Calculations with the Mean

1. The mean height of a class of 28 students is 162 cm. A new girl of height 149 cm joins the class. What is the mean height of the class now?

2. After 5 matches the mean number of goals scored by a football team per match is 1.8. If they score 3 goals in their 6th match, what is the mean after the 6th match?

3. The mean number of students ill at a school is 3.8 per day, for the first 20 school days of a term. On the 21st day 8 children are ill. What is the mean after 21 days?

4. The mean weight of 25 children in a class is 58 kg. The mean weight of a second class of 29 children is 62 kg. Find the mean weight of all the children.

5. A shop sells a mean of 4.6 fridges per day for 5 days. How many must it sell on the sixth day to increase his mean to 5 sales per day?

6. Uale's mean score for four tests is 64%. He wants to increase his mean to 68% after the fifth test. What does he need to score in the fifth test?

7. The mean salary of the 8 people who work for a small company is $15 000. When an extra worker is taken on this mean drops to $14 000. How much does the new worker earn?

8. The mean of 6 numbers is 12.3. When an extra number is added the mean changes to 11.9. What is the extra number?

9. When 5 is added to a set of 3 numbers the mean increases to 4.6. What was the mean of the original 3 numbers?

10. Three numbers have a mean of 64. When a fourth number is included the mean is doubled. What is the fourth number?

Mean, Median and Mode for Grouped Data

The mean and median can be estimated from tables of *grouped* data.

The class interval which contains the most values is known as the *modal class*.

Example 1

The table below gives data on the heights, in cm, of 51 children:

Class interval	Frequency
$140 \le h < 150$	6
$150 \le h < 160$	16
$160 \le h < 170$	21
$170 \le h < 180$	8

(a) Estimate the mean height.

(b) Estimate the median height.

(c) Find the modal class.

Solution

(a) To find the mean, the mid-point of each interval should be used.

Class Interval	Mid-point	Frequency	Mid-point × Frequency
$140 \le h < 150$	145	6	$145 \times 6 = 870$
$150 \le h < 160$	155	16	$155 \times 16 = 2480$
$160 \le h < 170$	165	21	$165 \times 21 = 3645$
$170 \le h < 180$	175	8	$175 \times 8 = 1400$
	Totals	51	8215

$$\text{Mean} = \frac{8215}{51}$$

$$= 161 \qquad \text{(to the nearest cm)}$$

(b) The median is the 26th value. In this case it lies in the $160 \le h < 170$ class interval. The 4th value in the interval is needed. It is estimated as

$$160 + \frac{4}{21} \times 10 = 162 \qquad \text{(to the nearest cm)}$$

(c) The modal class is $160 \le h < 170$ as it contains the most values.

Also note that when we speak of someone by age, say eight, then the person could be any age from 8 years 0 days up to 8 years 364 days (365 in a leap year!). You will see how this is tackled in the following example.

Example 2

The age of children in a primary school were recorded in the table below.

Age	5–6	7–8	9–10
Frequency	29	40	38

(a) Estimate the mean.

(b) Estimate the median.

(c) Find the modal age.

Solution

(a) To estimate the mean, we must use the mid-point of each interval; so, for example for '5–6', which really means

$5 \leq \text{age} < 7$,

the mid-point is taken as 6.

Class Interval	Mid-point	Frequency	Mid- point × Frequency
5–6	6	29	$6 \times 29 = 174$
7–8	8	40	$8 \times 40 = 320$
9–10	10	38	$10 \times 38 = 380$
	Totals	107	874

$$\text{Mean} = \frac{874}{107}$$

$$= 8.2 \qquad \text{(to 1 decimal place)}$$

(b) The median is given by the 54th value, which we have to estimate. There are 29 values in the first interval, so we need to estimate the 25th value in the second interval. As there are 40 values in the second interval, the median is estimated as being

$$\frac{25}{40}$$

of the way along the second interval. This has width $9 - 7 = 2$ years, so the median is estimated by

$$\frac{25}{40} \times 2 = 1.25$$

from the start of the interval. Therefore the median is estimated as $7 + 1.25 = 8.25$ years.

(c) The modal age is the 7–8 age group.

Example 1 uses what are called *continuous data*, since height can be of any value. (Other examples of continuous data are weight, temperature, area, volume and time.)

The next example uses *discrete data*, that is, data which can take only a particular value, such as the integers 1, 2, 3, 4 . . . in this case.

The calculations for mean and mode are not affected but estimation of the median requires the *discrete* grouped data with an approximate *continuous* interval.

Skill Exercises: Mean, Median and Mode for Grouped Data

1. A church minister keeps a record of the number of homes he visits each day.

Homes visited	0–9	10–19	20–29	30–39	40–49
Frequency	3	8	24	60	21

   (a) Estimate the mean number of homes visited.

   (b) Estimate the median.

   (c) What is the modal class?

2. The weights of a number of students were recorded in kg.

Mean (kg)	Frequency
$30 \leq w < 35$	10
$35 \leq w < 40$	11
$40 \leq w < 45$	15
$45 \leq w < 50$	7
$50 \leq w < 55$	4

   (a) Estimate the mean weight.

   (b) Estimate the median.

   (c) What is the modal class?

3. A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (seconds)	$10 \leq t < 15$	$15 \leq t < 20$	$20 \leq t < 25$	$25 \leq t < 30$
Frequency	6	16	21	8

   (a) Is the median in the modal class?

   (b) Estimate the mean.

   (c) Estimate the median.

   (d) Is the median greater or less than the mean?

4. The distances that students in a class travelled to school are recorded.

Distance (km)	Frequency
$0 \le d < 0.5$	30
$0.5 \le d < 1.0$	22
$1.0 \le d < 1.5$	19
$1.5 \le d < 2.0$	8

(a) Does the modal class contain the median?

(b) Estimate the median and the mean.

(c) Which is the largest, the median or the mean?

5. The ages of the children at a camp are summarised in the table below:

Age (years)	2–5	6–10	11–15	16–25
Frequency	20	26	32	5

Estimate the mean ages of the children.

6. The lengths of a number of leaves collected for a project are recorded.

Length (cm)	2–5	6–10	11–15	16–25
Frequency	8	20	42	12

Estimate (a) the mean (b) the median length of a leaf.

7. The table shows how many night tourists spend at a motel.

Number of nights	1–5	6–10	11–15	16–20	21–25
Frequency	20	26	32	5	2

(a) Estimate the mean.

(b) Estimate the median.

(c) What is the modal class?

8. (a) A teacher notes the number of correct answers given by a class in a multiple-choice test.

Correct answers	1–10	11–20	21–30	31–40	41–50
Frequency	2	8	15	11	3

(i) Estimate the mean.

(ii) Estimate the median.

(iii) What is the modal class?

(b) Another class took the same test. Their results are given below:

Correct answers	1–10	11–20	21–30	31–40	41–50
Frequency	3	14	20	2	1

(i) Estimate the mean.

(ii) Estimate the median.

(iii) What is the modal class?

(c) How do the results for the two classes compare?

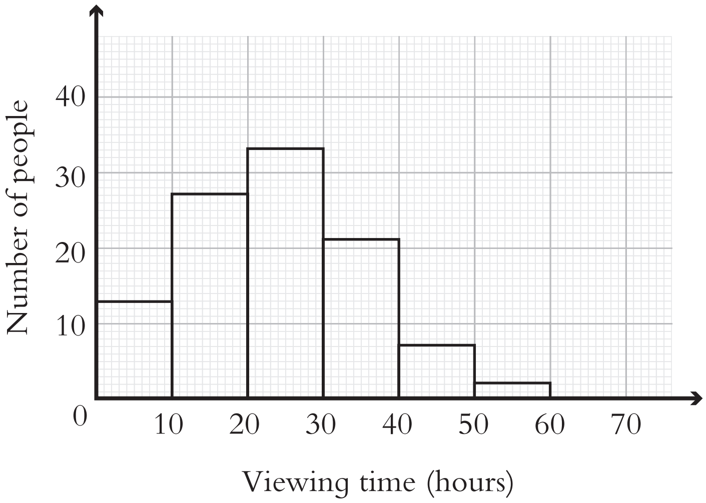
9. Twenty nine children are asked how much pocket money they were given last week. Their replies are shown in this frequency table.

Pocket Money $	Frequency f
0–$1.00	12
$1.01–$2.00	9
$2.01–$3.00	6
$3.01–$4.00	2

(a) Which is the modal class?

(b) Calculate an estimate of the mean amount of pocket money received per child.

10. The graph shows the number of hours a sample of people in New Zealand spent viewing television one week during the summer.



(a) Copy and complete the frequency table for this sample.

Viewing time (h hours)	Number of people
$0 \le h < 10$	13
$10 \le h < 20$	27
$20 \le h < 30$	33
$30 \le h < 40$	
$40 \le h < 50$	
$50 \le h < 60$	

(b) Another survey is carried out during the winter. State one difference you would expect to see in the data.

(c) Use the mid-points of the class intervals to calculate the mean viewing time for these people. You may find it helpful to use the table below.

Viewing time (h hours)	Mid-point	Frequency	Mid-point × Frequency
$0 \le h < 10$	5	13	65
$10 \le h < 20$	15	27	405
$20 \le h < 30$	25	33	825
$30 \le h < 40$	35		
$40 \le h < 50$	45		
$50 \le h < 60$	55		

Cumulative Frequency

Cumulative frequencies are useful if more detailed information is required about a set of data. In particular, they can be used to find the median and inter-quartile range.

The *inter-quartile range* contains the middle 50% of the sample and describes how spread out the data are. This is illustrated in Example 2.

Example 1

For the data given in the table, draw up a cumulative frequency table and then draw a cumulative frequency graph.

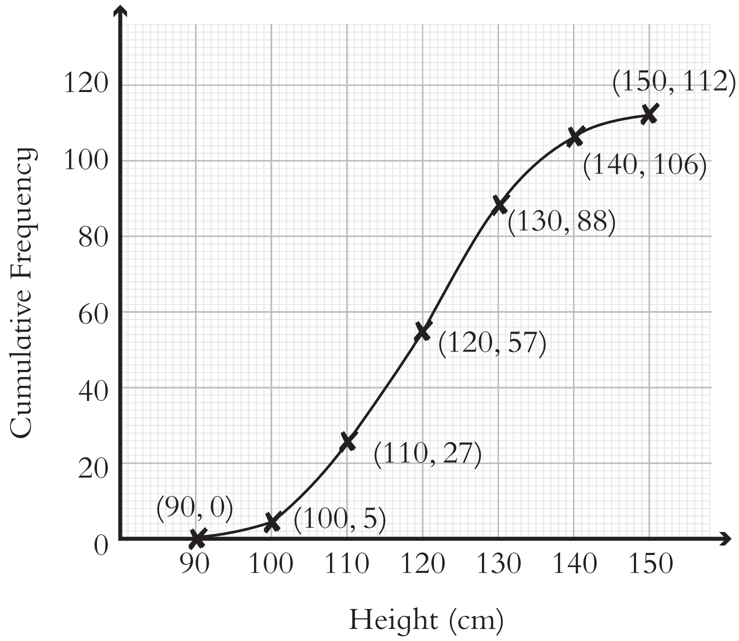
Height (cm)	Frequency
$90 < h \le 100$	5
$100 < h \le 110$	22
$110 < h \le 120$	30
$120 < h \le 130$	31
$130 < h \le 140$	18
$140 < h \le 150$	6

Solution

The table below shows how to calculate the cumulative frequencies.

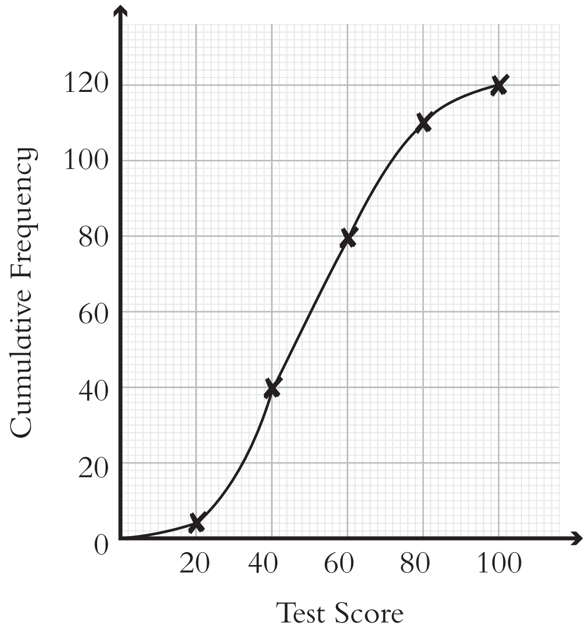
Height (cm)	Frequency	Cumulative Frequency
$90 < h \le 100$	5	5
$100 < h \le 110$	22	$5 + 22 = 27$
$110 < h \le 120$	30	$27 + 30 = 57$
$120 < h \le 130$	31	$57 + 31 = 88$
$130 < h \le 140$	18	$88 + 18 = 106$
$140 < h \le 150$	6	$106 + 6 = 112$

A graph can then be plotted using points as shown below:



Example 2

The cumulative frequency graph below gives the results of 120 students on a test.



Use the graph to find:

(a) the median score,

(b) the inter-quartile range,

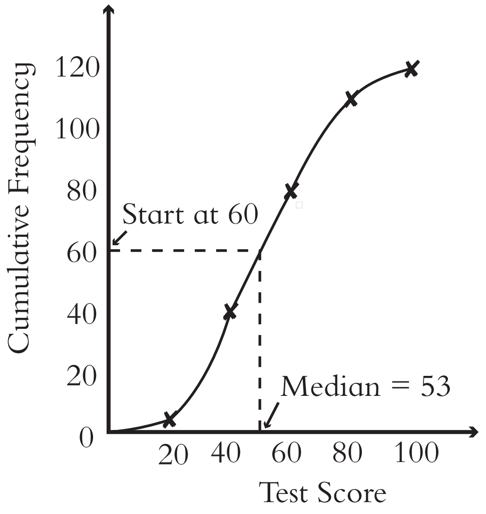
(c) the mark which was attained by only 10% of the students,

(d) the number of students who scored more than 75 on the test.

Solution

(a) Since $\frac{1}{2}$ of 120 is 60, the *median* can be found by starting at 60 on the vertical scale, moving horizontally to the graph line and then moving vertically down to meet the horizontal scale.

In this case the median is 53.



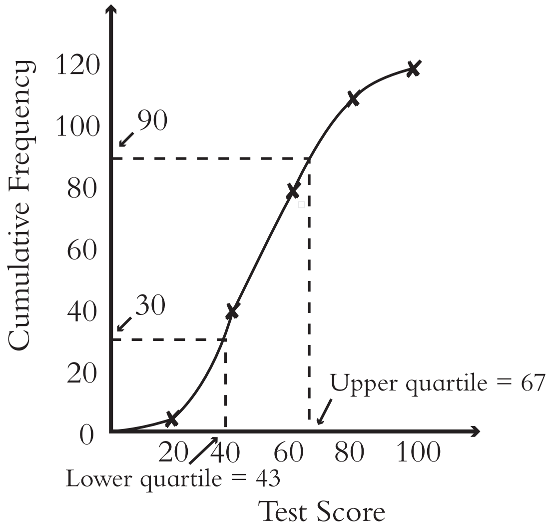
(b) To find out the *inter-quartile range*, we must consider the middle 50% of the students.

To find the *lower quartile*, start at $\frac{1}{4}$ of 120, which is 30.

This gives
Lower Quartile = 43.

To find the *upper quartile*, start at $\frac{3}{4}$ of 120, which is 90.

This gives
Upper Quartile = 67.



The *inter-quartile range* is then

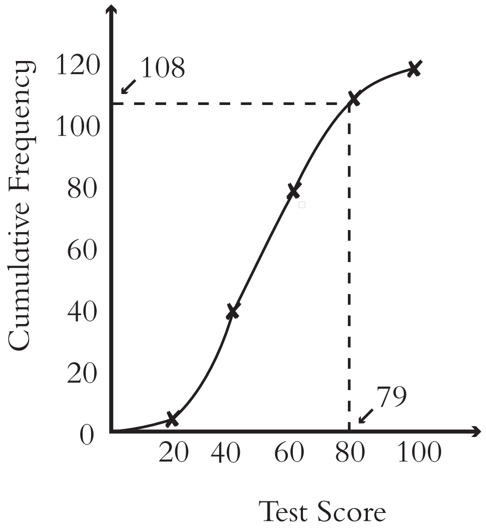
Inter-quartile Range = Upper Quartile – Lower Quartile
= 67 – 43
= 24.

(c) Here the mark which was obtained by the top 10% is required.

10% of 120 = 12

so start at 108 on the cumulative frequency scale.

This gives a mark by 79.

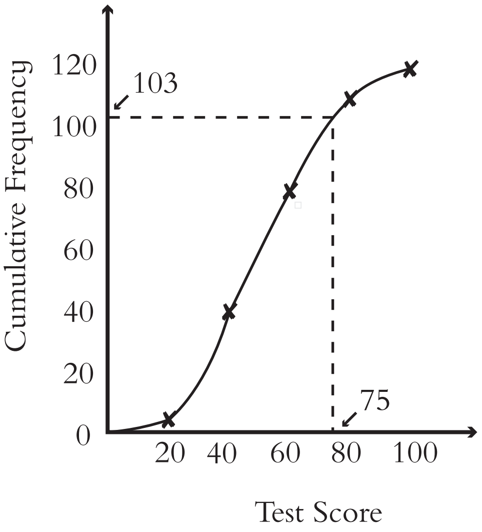


(d) To find the number of students who scored more than 75, start at 75 on the horizontal axis.

This gives a cumulative frequency of 103.

So the number of students with a score greater than 75 is

120 – 103 = 17.



Skill Exercises: Cumulative Frequency

1. Make a cumulative frequency table for each set of data given below. Then draw a cumulative frequency graph and use it to find the median and inter-quartile range.

(a) Ioane weighed each mango in a large box. His results are given in this table.

Weight of apple (g)	Frequency
$60 < w \leq 80$	4
$80 < w \leq 100$	28
$100 < w \leq 120$	33
$120 < w \leq 140$	27
$140 < w \leq 160$	8

(b) Pasi asked the students in his class how far they travelled to school each day. His results are given below.

Distance (km)	Frequency
$0 < d \leq 1$	5
$1 < d \leq 2$	12
$2 < d \leq 3$	5
$3 < d \leq 4$	6
$4 < d \leq 5$	5
$5 < d \leq 6$	3

(c) A Physical Education teacher recorded the distances students could reach in the long jump event. His records are summarised in the table below.

Length of jump (m)	Frequency
$1 < d \leq 2$	5
$2 < d \leq 3$	12
$3 < d \leq 4$	5
$4 < d \leq 5$	6
$5 < d \leq 6$	5

2. A farmer grows a type of lettuce in two different fields. He takes a sample of 50 lettuces from each field at random and weighs the plants he gets.

Mass of lettuce (g)	Frequency Field A	Frequency Field B
$0 < m \leq 5$	3	0
$5 < m \leq 10$	8	11
$10 < m \leq 15$	22	34
$15 < m \leq 20$	10	4
$20 < m \leq 25$	4	1
$25 < m \leq 30$	3	0

(a) Draw cumulative frequency graphs for each field.

(b) Find the median and inter-quartile range for each field.

(c) Comment on your results.

3. A consumer group tests two types of batteries using a personal stereo.

Lifetime (hours)	Frequency Type A	Frequency Type B
$2 < l \leq 3$	1	0
$3 < l \leq 4$	3	2
$4 < l \leq 5$	10	2
$5 < l \leq 6$	22	38
$6 < l \leq 7$	8	6
$7 < l \leq 8$	4	0

(a) Use cumulative frequency graphs to find the median and inter-quartile range for each type of battery.

(b) Which type of battery would you recommend and why?

4. The table below shows how the height of girls of a certain age vary.The data was gathered using a large-scale survey.

Height (cm)	Frequency
$50 < h \leq 55$	100
$55 < h \leq 60$	300
$60 < h \leq 65$	2400
$65 < h \leq 70$	1300
$70 < h \leq 75$	700
$75 < h \leq 80$	150
$80 < h \leq 85$	50

A doctor wishes to quantify children as:

Category	Percentage of Population
Very Tall	5%
Tall	15%
Normal	60%
Short	15%
Very short	5%

Use a cumulative frequency graph to find the heights of children in each category.

5. The manager of a glass company employs 30 salesman. Each year he awards bonuses to his salesmen.

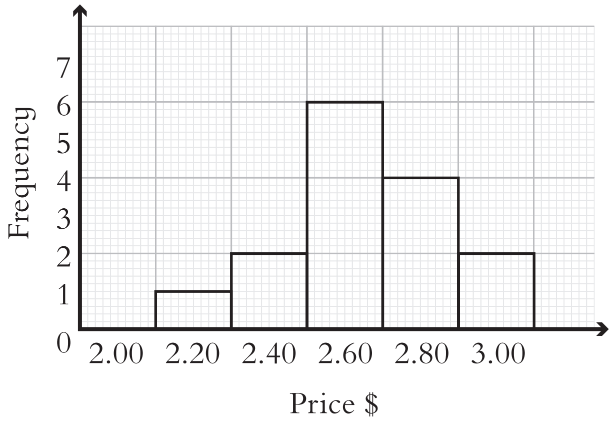
Bonus	Awarded to
\$500	Best 10% of salesmen
\$250	Middle 70% of salesmen
\$50	Bottom 20% of salesmen

The sales made during 2000 and 2001 are shown in the table below:

Value of sales (\$1000)	Frequency 2001	Frequency 2000
$0< V \leq 100$	0	2
$100 < V \leq 200$	2	8
$200 < V \leq 300$	15	18
$300 < V \leq 400$	10	2
$400 < V \leq 500$	3	0

Use cumulative frequency graphs to find the values of sales needed to obtain each bonus in the years 2000 and 2001.

6. The histogram shows the cost of buying a bag of rice in a number of different shops.



(a) Draw a cumulative frequency graph and use it to answer the following questions:

(i) How many shops charged more than \$2.65?

(ii) What is the median price?

(iii) How many shops charged less than \$2.30?

(iv) How many shops charged between \$2.20 and \$2.60?

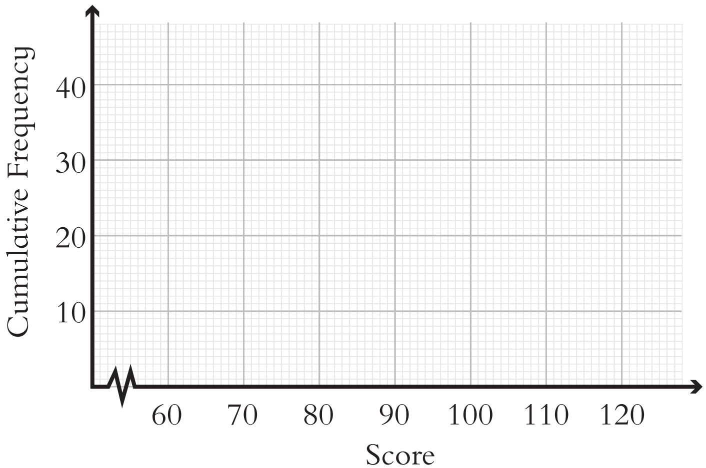
(v) How many shops charged between \$2.00 and \$2.50?

(b) Comment on which of your answers are exact and which are estimates.

7. Laura and Simiona played 40 games of golf together. The table below shows Laura's scores:

Scores (x)	Frequency
$70 < x \leq 80$	1
$80 < x \leq 90$	4
$90 < x \leq 100$	15
$100 < x \leq 110$	17
$110 < x \leq 120$	3

(a) On a grid similar to the one below, draw a cumulative frequency diagram to show Laura's scores.



(b) Making your method clear, use your graph to find:

 (i) Laura's median score

 (ii) The inter-quartile range of her scores.

(c) Simiona's median score was 103. The inter-quartile range of her scores was 6.

 (i) Who was the more consistent player? Give a reason for your choice.

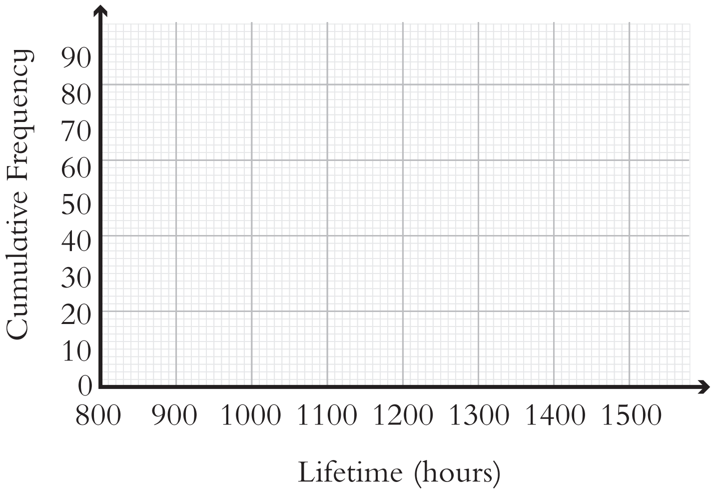
 (ii) The winner of a game of golf is the one with the lowest score. Who won most of these 40 games? Give a reason for your choice.

8. A sample of 80 light bulbs is shown. The lifetime of each light bulb was recorded. The results are shown below.

Lifetime (hours)	Frequency	Cumulative Frequency
800–	4	4
900–	13	17
1000–	17	
1100–	22	
1200–	20	
1300–	4	
1400–	0	

(a) Copy and complete the table of values for the cumulative frequency.

(b) Draw the cumulative frequency curve, using a grid as shown below:



(c) Use your graph to estimate the number of light bulbs which lasted more than 1030 hours.

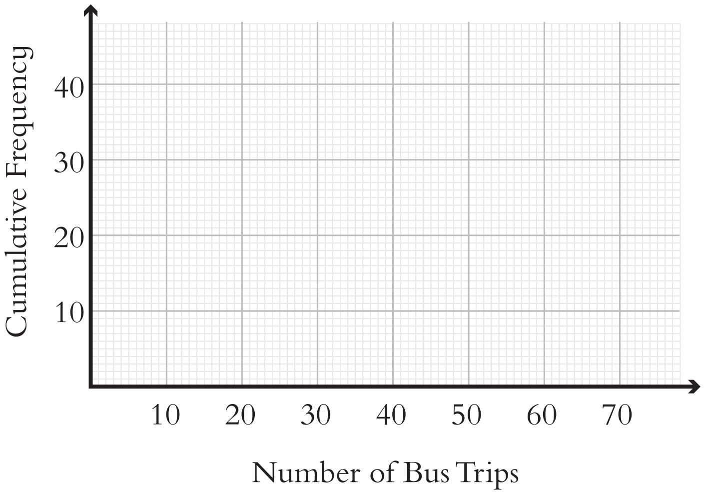
(d) Use your graph to estimate the inter-quartile range of the lifetimes of the light bulbs.

(e) A second sample of 80 light bulbs has the same median lifetime as the first sample. Its inter-quartile range is 90 hours. What does this tell you about the difference between the two samples?

9. The numbers of journeys made by a group of people using buses in one month are summarised in the table:

Number of journeys	Number of people
0–10	4
11–20	7
21–30	8
31–40	6
41–50	3
51–60	4
61–70	0

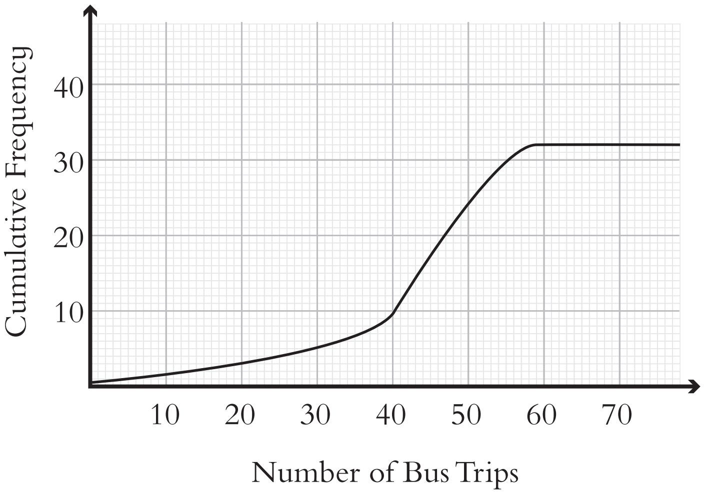
(a) Draw the cumulative frequency graph, using a grid as below:



(b) Use your graph to estimate the median number of bus trips.

(c) Use your graph to estimate the number of people who made more than 44 bus trips in the month.

(d) The numbers of journeys made using buses in one month, by another group of people, are shown in the graph.



Make one comparison between the numbers of journeys made by these two groups.

Unit 8: PROBABILITY

In this unit you will be:

8.1 Calculating Probabilities

- Simple Probability.
- Determining the Outcomes of Two Events.
- Finding Probabilities Using Relative Frequency.
- Finding the Theoretical Probability of One Event.
- Finding the Theoretical Probability of Two Events.
- Using Tree Diagrams.

Section 8.1 Calculating Probabilities

Simple Probability

Probabilities are given values between 0 and 1. A *probability* of 0 means that the event is *impossible*, while a probability of 1 means that that it is *certain*. The closer the probability of an event is to 1, the more likely it is to happen. The closer the probability of an event to 0, the less likely it is to happen.

Example 1

When you toss a coin, what is the probability that it lands heads up?

Solution

When you toss a coin there are two possibilities, that it lands heads up or tails up. As one of these must be obtained,

$$p(\text{heads}) + p(\text{tails}) = 1$$

But both are equally likely so

$$p(\text{heads}) = p(\text{tails}) = \frac{1}{2}.$$

Example 2

The probability that it rains tomorrow is $\frac{2}{3}$.

What is the probability that it does not rain tomorrow?

Solution

Tomorrow it must either rain or not rain, so

$$p(\text{rain}) + p(\text{no rain}) = 1.$$

The probability that it rains is $\frac{2}{3}$, so

$$\begin{aligned} \frac{2}{3} + p(\text{no rain}) &= 1 \\ p(\text{no rain}) &= 1 - \frac{2}{3} \\ &= \frac{1}{3}. \end{aligned}$$

So the probability that it does not rain is $\frac{1}{3}$.

Skill Exercises: Simple Probability

1. What is the probability that it does not rain tomorrow, if the probability that it does rain tomorrow is:

   (a) 0.9 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$?

2. Ben plays snooker with his friends. The probability that he beats Perenise is 0.8 and the probability that he beats Matiu is 0.6.

   (a) What is the probability that Perenise beats Ben?

   (b) What is the probability that Matiu beats Ben?

3. The probability that a bus is late arriving at its destination is 0.02. What is the probability that it is not late?

4. Iosefa has a trick coin. When he tosses it the probability of getting a head is $\frac{1}{5}$. What is the probability of getting a tail?

5. A weather forecaster states that the probability that it will rain tomorrow is $\frac{3}{7}$.

   (a) Find the probability that it does not rain tomorrow.

   (b) Is it more likely to rain or not to rain tomorrow?

6. The probability that it will snow during the winter in Seattle, USA is 0.01. What is the probability that it does not snow?

7. A school basketball team play 20 matches each year. The probability that they win any match is $\frac{3}{5}$.

   (a) What is the probability that they lose a match?

   (b) How many matches can they expect to win each year?

8. When Kereti plays battle chess on her computer the probability that she wins depends on the level at which she plays the game.

Level	Probability Kereti wins
Easy	0.9
Medium	0.4
Hard	0.1

   What is the probability that the computer wins if the level is set to:

   (a) Medium (b) Hard (c) Easy?

9. A student is selected at random from a school. The probability the child is a girl is $\frac{11}{20}$, the probability that the child is left handed is $\frac{1}{11}$ and the probability that the child wears glasses is $\frac{4}{13}$.

   Find the probabilities that a child selected at random,

   (a) is a boy (b) is right handed (c) does not wear glasses.

10. It has been estimated that in France the probability that a person has blue eyes is $\frac{4}{9}$.

    Is it true that the probability that a person has brown eyes is $\frac{5}{9}$?

11. A machine makes compact discs. The probability that a perfect compact disc will be made by this machine is 0.85.

    Work out the probability that a compact disc made by this machine will not be perfect.

12. Here are three possible events:

    A A coin when tossed will come down heads.

    B It will snow in August in Samoa.

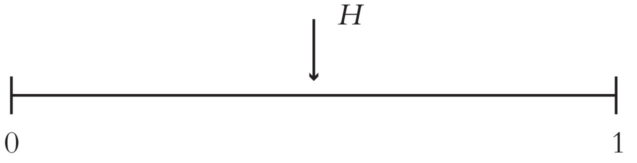
    C There will be a baby born tomorrow.

    Which of the three events is:

    (a) Most likely to happen?

    (b) Least likely to happen?

13.



A probability line is shown above. The arrow H on the line shows the possibility that, when a coin is tossed, it will come down 'heads'.

(a)

Copy the probability line and put an arrow S on the line to show the probability that it will snow where you live tomorrow.

(b) Put an arrow, L on the line to show the probability that the next truck you see travelling on the road will have a male driver. Explain why you put your arrow in that position.

Determining the Outcomes of Two Events

When dealing with probabilities for two events, it is important to be able to identify all the possible outcomes. Here are examples to show the methods that can be used.

Method A: Systematic Listing

Example 1

For a special meal customers at a pizza parlour can choose a pizza with *one* of the following toppings:

Ham	Mushroom	Salami
Pepperoni	Tuna	

and a drink from the following list:

Cola	Diet Cola	Orange

How many possible combinations of toppings and drinks are there?

Solution

Using the first letter of each drink and topping, it is easy to see that Cola (C) could be combined with any of the five toppings to give CH, CM, CS, CP, CT. Here 'CH' means 'Cola' drink and 'Ham' topping, etc.

Similarly, for Diet Cola (D), you have

DH, DM, DS, DP, DT

And for Orange (O)

OH, OM, OS, OP, OT

You can see that there are $3 \times 5 = 15$ possible outcomes.

This method of listing will always work but it might be slow, particularly if there are more than 2 choices to be made.

Method B: 2-Way Tables

Example 2

A six-sided die and a coin are tossed. List all the possible outcomes.

Solution

The coin can land heads (denoted by H) or tails (T), whilst the die can show 1, 2, 3, 4, 5 or 6. So for heads on the coin, the possible outcomes are:

H1, H2, H3, H4, H5 and H6.

Whilst for tails, they are:

T1, T2, T3, T4, T5 and T6.

The listing method used here can be conveniently summarised in a 2-way table.

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

This method works well but cannot be used if there are more than 2 choices to be made.

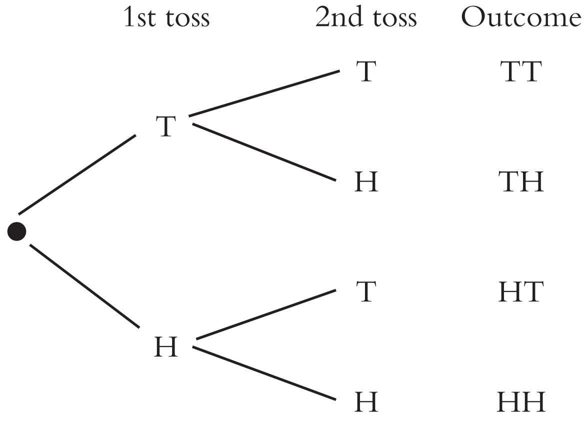
Method C: Tree Diagrams

Example 3

A coin is tossed twice. List all the possible outcomes.

Solution

You can use a tree diagram to represent this solution.



Note that 'TH' is not the same as 'HT'.

This is an excellent method, but can lead to problems if you have too many branches.

Skill Exercises: Determining the Outcomes of Two Events

1. Three flavours of ice cream, vanilla (V), mint (M), and raspberry ripple (R), are available at a shop. Each is served with a topping of either chocolate (C) or strawberry (S).

   One possible order is for vanilla ice cream with chocolate topping (VC).

   Write a list of all the other possibilities.

2. A bag contains two balls which are the same size. One is green and one is red. You take a ball out of the bag, put it back, then take another.

   Make a list of all the possible outcomes for the colours of the two balls.

3. Three boys, Ben, Ioane and Roy, decide to hold a competition. They will do sit-ups and then press-ups.

   If Ben wins the sit-ups and Ioane wins the press-ups, the outcome would be represented as BI.

   (a) What does RB represent?

   (b) Make a list of all the 9 possible outcomes.

   (c) If only Ben and Ioane take part in the competition there will be fewer possible outcomes. List the outcomes in this case.

   (d) If Timoti also takes part in the competition, list all the possible outcomes for the four competitors.

4. Packets of cornflakes contain a free model dinosaur. There are four different models, the Brontosaurus (B), the Stagosaurus (S), Tyrannosaurus-Rex (T) and Diplodocus (D). A mother buys two packets of cornflakes for her children. List all combinations of free gifts possible when the packets are opened.

5. At a school Christmas Fair three different sorts of prizes can be won in a lucky dip. One is a cassette tape (C), one is a diary (D), and the other a book (B).

   List all the possible outcomes for a girl who has two goes at the lucky dip.

6. For breakfast, Rachel will drink either fruit juice (F) or cold milk (M) and will eat cornflakes (C), honey-crunch loops (H) or toast (T).

   Complete a copy of the table below to show the possible outcomes for her choice of breakfast.

Drinks

		F	M
Food	H		
	T		
	C		

Finding Probabilities Using Relative Frequency

Sometimes it is possible to calculate values for the probability of an event by symmetry arguments, like tossing a coin and getting a head. For other events probabilities can be estimated by using results of experiments or records of past events.

Example 1

In February 2000 it rained on 18 days. Use this information to estimate the probability that it rains on a day in February.

Solution

It rained 18 out of the 28 days, so the relative frequency of rain is

$$\frac{18}{28} = \frac{9}{14}.$$

So the probability that it rains can be estimated as $\frac{9}{14}$.

Example 2

Sita carries out an experiment with a piece of buttered toast. She drops it 50 times and finds out that 35 times it lands butter side down. Use these results to estimate the probability that a piece of toast lands butter side down when dropped.

Solution

The toast landed butter side down 35 of the 50 times,
so the relative frequency is $\frac{35}{50} = \frac{7}{10}$.

So the probability that the toast lands butter side down can be estimated as $\frac{7}{10}$.

Skill Exercises: Finding Probabilities Using Relative Frequency

1. (a) Conduct an experiment with a drawing pin, by dropping it in the same way a large number of times. You could drop it 100 times and record whether it lands point up or point down.

   (b) Use your results to estimate the probability that a drawing pin lands point up.

2. (a) Obtain a short stick, such as a toothpick. On a sheet of A4 paper draw parallel lines that are 6 cm apart. Drop the stick onto a sheet of paper a large number of times and record whether or not it lands on a line.

   (b) Use your results to estimate the probability that the stick lands on a line.

3. When you toss a coin you would expect to get a head half of the time.
   (a) Toss a coin 20 times and record the results. How well do they compare with your expectation.
   (b) Toss the coin another 30 times, so that you have 50 results. How well do they compare with your expectation now?

4. Anatalia observed that the school bus was late on 6 of the 24 school days in March. Estimate the probability that the bus was late on any one day.

5. A football team plays on average 40 matches each season and wins 32 of them. Estimate the probability that this team wins a match.

6. Six students play regularly in a chess club. The number of games that each student has won is recorded in the table below.

Player	Games Won	Games Lost
Timoti	4	10
Anatalia	7	3
Tanielu	3	9
Rachel	4	16
Siaki	6	12
Malia	12	6

   (a) Use this data to find the probability that each child wins a match.
   (b) Which child is the best player?
   (c) Which child is the worst player?
   (d) If Siaki played Timati, who do you think would be most likely to win?

7. A car yard records the number of cars that they sell each week over a 24 week period. The numbers for each week are given below.

   3, 4, 8, 6, 5, 7, 4, 3, 6, 5, 2, 4,

   5, 7, 6, 9, 2, 4, 5, 6, 7, 4, 3, 5.

   Use this data to establish the probability that in any week;
   (a) more than 5 cars are sold.
   (b) fewer than 5 cars are sold.
   (c) exactly 5 cars sold.

8. A gardener plants 40 seeds and 32 of them produce healthy plants.
   (a) Estimate the probability that a seed produces a healthy plant.
   (b) If 120 seeds were planted, how many healthy plants can the gardener expect to obtain?

Finding the Theoretical Probability of One Event

When the outcomes of an event are all equally likely, then probabilities can be found by considering all the possible outcomes.

For example, when you toss a coin of two possible outcomes, either heads or tails.

So $$p(\text{head}) = \frac{1}{2}$$

$$p(\text{tails}) = \frac{1}{2}$$

The probability of an outcome is given by

$$\frac{\text{number of ways of obtaining outcome}}{\text{number of possible outcomes}}$$

provided all the outcomes are equally likely.

Example 1

A card is taken at random from a full pack of playing cards with no jokers. What is the probability that the card:

(a) is an ace?

(b) is black?

(c) is a heart?

(d) has an even number on it?

Solution

First note that each card is equally likely to be selected, and that there are 52 possible outcomes.

(a) There are four aces, so

$$p(\text{ace}) = \frac{4}{52} = \frac{1}{13}$$

(b) There are 26 black cards, so

$$p(\text{black}) = \frac{26}{52} = \frac{1}{2}$$

(c) There are 13 hearts in the pack, so;

$$p(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

(d) There are 20 cards with even numbers on them, so;

$$p(\text{even number}) = \frac{20}{52} = \frac{5}{13}$$

Example 2

In a class of 30 students, 16 are girls, 4 wear glasses and 3 are left-handed.

A child is chosen at random from the class. What is the probability that the child is:

(a) a girl? (b) right-handed? (c) wearing glasses?

Solution

All the children in the class are equally likely to be selected, when the choice is made at random.

(a) In the class there are 16 girls, so

$$p(\text{girl}) = \frac{16}{30} = \frac{8}{15}$$

(b) There are 3 left-handed children and so the other 27 must be right-handed. So,

$$p(\text{right-handed}) = \frac{27}{30} = \frac{9}{10}$$

(c) There are 4 children wearing glasses. So,

$$p(\text{wears glasses}) = \frac{4}{30} = \frac{2}{15}$$

Skill Exercises: Finding the Theoretical Probability of One Event

1. Riki takes a card at random from a full pack of playing cards. What is the probability that his card:

   (a) is a diamond? (b) is a spade? (c) is a seven?

   (d) is a king? (e) has a prime number on it?

2. Repeat question 1, for a pack of playing cards containing 2 jokers (a total of 54 cards).

3. When you roll an ordinary dice, what is the probability of obtaining:

   (a) a 6? (b) a 5? (c) an even number?

   (d) a prime number?

4. A new game includes an octagonal roller with faces numbered from 1 to 8. When the roller is rolled, what is the probability of obtaining:

   (a) an 8?
   (b) a 1?
   (c) an odd number?
   (d) a number greater than 3?
   (e) a number less than 3?

5. In a class of 32 students, 20 have school lunches and the rest bring sandwiches. What is the probability that a child chosen at random from the class brings sandwiches?

6. In a lucky dip at a school fair, a tub contains 50 prizes at the start of the fair. There are 20 superballs, 10 pens, 10 toy cars and 10 packets of sweets.

   What is the probability that the first person to visit the lucky dip:

   (a) wins a superball?
   (b) does not win a pen?
   (c) wins a packet of sweets?
   (d) does not win a toy car?

   If the first person wins a pen, what is the probability that the second person:

   (e) wins a pen?
   (f) does not win a toy car?
   (g) wins a packet of sweets?

7. A bus leaves Faleolo airport with 18 passengers. It stops at Faleasi'u, where another 12 passengers join the coach. At Fale'ula it stops again and 20 more passengers get on board. When the bus arrives at its destination all the passengers get off and one is chosen at random to be interviewed about the journey.

   Find the probabilities that this passenger:

   (a) was on the bus for the whole journey.
   (b) got on the bus at Faleasi'u.
   (c) got on the bus at Faleasi'u or Faleolo.
   (d) got on the bus at Falaeasi'u or Fale'ula.

8. Five different types of model dinosaurs are being given away in cornflakes packets. A model dinosaur is put into each packet at random and five dinosaurs are needed for a complete set.

   (a) Ben already has 3 of the 5 models. What is the probability he gets a different one in the next packet he opens?
   (b) Atama only needs one more dinosaur to complete his set. What is the probability that he gets this dinosaur in the next packet he opens?
   (c) Ioane has only one dinosaur in his collection. What is the probability that he gets the same one in his next packet?

9. A bag contains 5 red counters, 3 green counters and 2 blue counters. Counters are taken out of the bag at random, but are not put back into the bag.
   (a) What is the probability that the first counter taken out is green?
   (b) If the first counter is green, what is the probability that the second counter is green?
   (c) If the first two counters are green, what is the probability that the third counter is green?
   (d) If a red counter is followed by a blue counter, what is the probability that the third counter is green?

Finding the Theoretical Probability of Two Events

When two events take place, and every outcome is equally likely to happen, the probability of a particular combined outcome can be readily found from the formula

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

The next examples show how this formula is used.

Example 1

Two dice are thrown together. Find the probability that the total score is 9.

Second Die

First Die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	⑨
4	5	6	7	8	⑨	10
5	6	7	8	⑨	10	11
6	7	8	⑨	10	11	12

Solution

The table shows all the possible outcomes and total scores.

There are 36 possible outcomes, and each one is equally likely to occur.

The outcomes that give a total of 9 have been circled. The are 4 such outcomes.

Now the probability can be found.

$$p(9) = \frac{4}{36} = \frac{1}{9}$$

Example 2

A spinner which forms part of a children's game can point to one of four regions A, B, C or D, when spun. What is the probability that when two children spin the spinner, it points to the same letter?

Solution

The table shows all the possible outcomes:

Second Child

First Child

	A	B	C	D
A	AA	AB	AC	AD
B	BA	BB	BC	BD
C	CA	CB	CC	CD
D	DA	DB	DC	DD

There are 16 possible outcomes. Each is equally likely to occur. The outcomes that are the same for both children have been circled. There are four outcomes of this type.

The probability that both have the same letter is

$$\frac{4}{16} = \frac{1}{4}$$

Note: It is expected that fractions are used for expressing probabilities, but using decimals is equally acceptable.

Skill Exercises: Finding the Theoretical Probability of Two Events

1. When two coins are tossed together the possible outcomes are as shown in the table.

Second Coin

First Coin

	H	T
H	HH	HT
T	TH	TT

(a) What is the probability that both coins show heads?

(b) What is the probability that only one coin shows a tail?

(c) What is the probability that both coins land the same way up?

2. A coin is tossed and a die is rolled. Copy and complete the table below to show the possible outcomes.

Die

Coin		1	2	3	4	5	6
	H	H1					
	T						

What is the probability of obtaining:

(a) a head and a 6?

(b) a tail and an odd number?

(c) a head and an even number?

(d) a head and a number greater than 2?

(e) an even number?

3. (a) Use this table, which shows the outcomes when two dice are rolled, to find the probabilities of each event described below:

Second Die

First Die		1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(i) a score of 7.

(ii) a score of 5.

(iii) a score that is an even number.

(iv) a score of more than 8.

(v) a score of less than 6.

(b) What score are you most likely to get when you roll *two* dice?

4. A cook decides at random what flavour drink to serve at lunch time. She chooses from pineapple, orange and lemon.

(a) Complete a copy of the table to show the possible outcomes from two consecutive days.

Second Day

First Day		P	L	O
	P	PP		
	L			LO
	O			

(b) What is the probability that she serves:

(i) pineapple on both days?

(ii) the same flavour on both days?

(iii) lemon or pineapple on both days?

(c) Kala is allergic to lemon. What is the probability that she is unable to have the drink on two consecutive days?

5. A young couple decide that they will have two children. There is an equal chance that each child will be a boy or girl.
   (a) Find the probability that both children are boys.
   (b) Find the probability that both children are of the same sex.

6. A game contains two tetrahedral dice which have 4 faces numbered 1 to 4.

   The two dice are thrown, and the total score is noted.
   (a) Find the probability that a score of 3 is obtained.
   (b) Find the probability of getting a score of more than 4.
   (c) Which score is most likely?

7. A bag contains one red ball, one blue ball and one green ball. One ball is taken out of the bag. A second ball is also taken out, without replacing the first ball. The table shows the possible outcomes.
   (a) Explain why some entries in the table have been marked with an X. How many possible outcomes are there?
   (b) What is the probability that the red ball is selected?
   (c) What is the probability that the green ball is left in the bag?

Second Ball

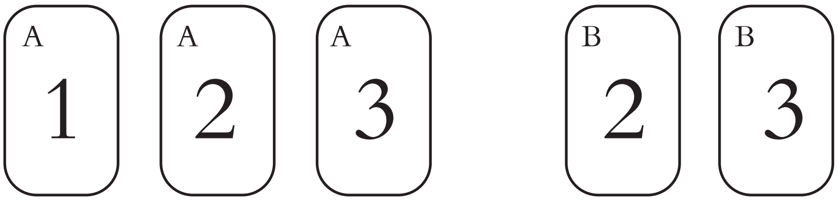
First Ball

	R	B	G
R	X	RB	RG
B	BR	X	BG
G	GR	GB	X

8. Two darts are thrown at a dartboard so that they are equally likely to hit any number between 1 and 20. Ignore doubles, trebles and the bull's-eye.
   (a) How many possible outcomes are there?
   (b) What is the probability of a score of 2?
   (c) What is the probability that both darts hit the same number?
   (d) What is the probability of a score of 17?

9. Three coins are tossed at the same time. Find the probabilities that:
   (a) they all land the same way up.
   (b) they all land with heads showing.
   (c) at least one coin lands showing tails.

10. The diagram shows two sets of cards A and B.



(a) One card is chosen at random from set A. One card is chosen at random from set B.

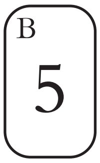
(i) List all the possible outcomes.

The two numbers are added together.

(ii) What is the probability of getting a total of 5?

(iii) What is the probability of getting a total that is not 5?

A new card is added to the set B. It is:



(b) (i) How many possible outcomes are there now?

(ii) Explain why adding the new card *does not* change the number of outcomes that have a total of 5.

(iii) Explain why adding the new card *does* change the probability of getting a total of 5.

Using Tree Diagrams

Tree diagrams can be used to find the probabilities for two events, when the outcomes are not necessarily equally likely.

Example 1

If the probability that it rains on any day is $\frac{1}{5}$, draw a tree diagram and find the probability that:

(a) it rains on two consecutive days.

(b) it rains on only one of two consecutive days.

Solution

The tree diagram shows all the possible outcomes. Then the probability of each event can be placed on the appropriate branch of the tree. The probability of no rain is $1 - \frac{1}{5} = \frac{4}{5}$.

The probability of each outcome is obtained by multiplying together the probabilities on the branches leading to that outcome. For rain on the first day, but not on the second, the probability is:

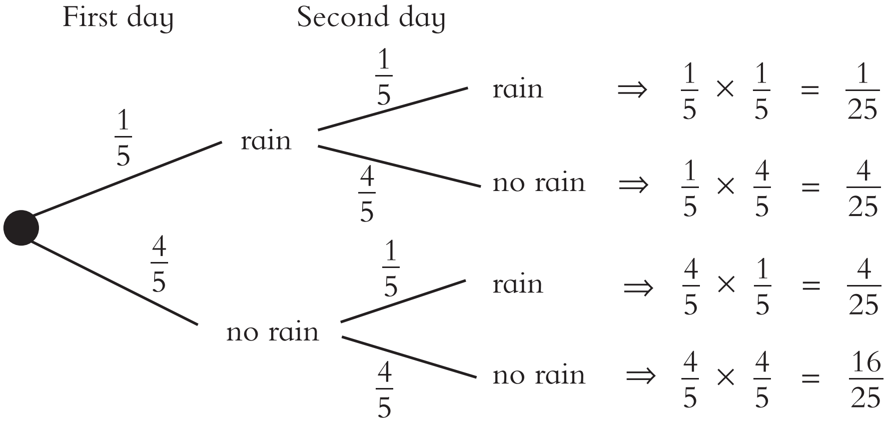
$$\frac{1}{5} \times \frac{4}{5} = \frac{4}{25}$$

(a) The probability that it rains on two consecutive days is given by the top set of branches, and is $\frac{1}{25}$.

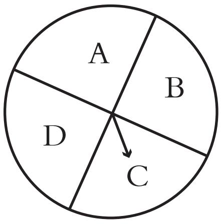
(b) There are two outcomes where there is rain on only one of two days. These are rain – no-rain, with a probability of $\frac{4}{25}$ and no-rain – rain with a probability of $\frac{4}{25}$.

The probability of rain on only one day is found by adding these two probabilities together:

$$\frac{4}{25} + \frac{4}{25} = \frac{8}{25}$$



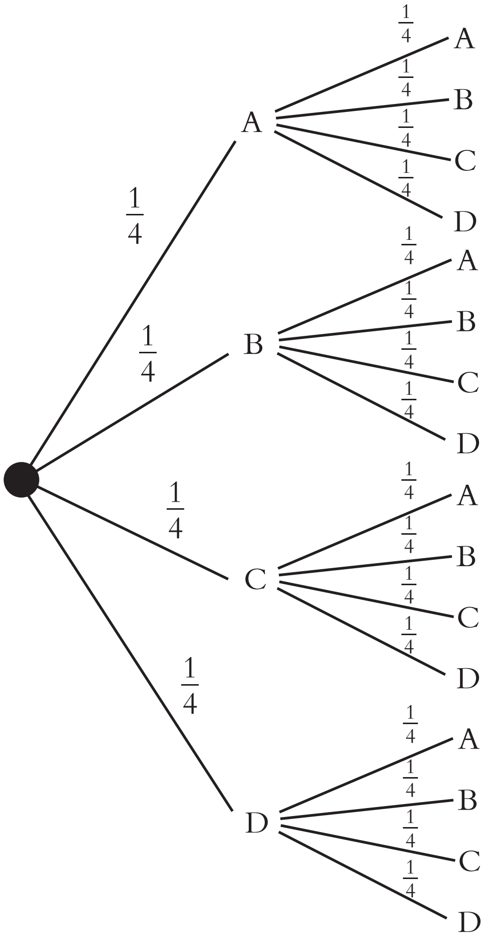
Example 2



A spinner forms part of a children's game can point to one of four regions, A, B, C or D, when spun. What is the probability that when two children spin the spinner, it points to the same letter?

Solution

This time, let us use the tree diagram approach.



$\Rightarrow p(AA) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$\Rightarrow p(BB) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$\Rightarrow p(CC) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$\Rightarrow p(DD) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

So the probability of both children obtaining the same letter is:

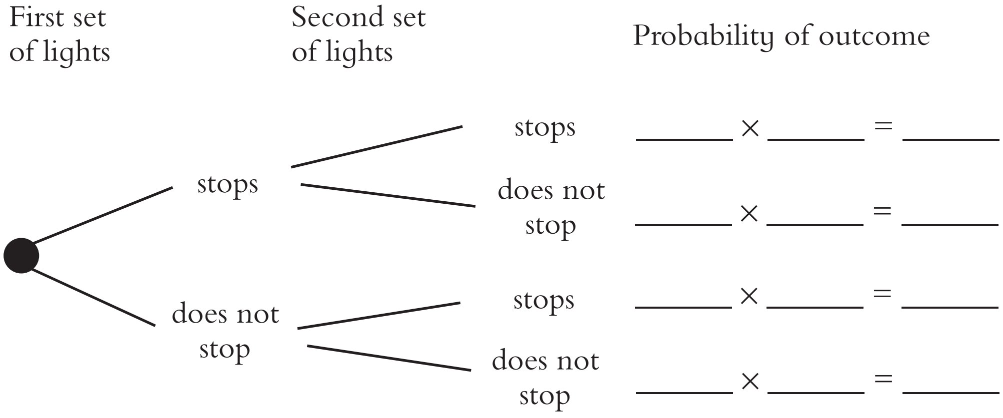
$$\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4} \quad \text{(as obtained before)}$$

Skill Exercises: Using Tree Diagrams

1. On a route to school a bus must pass through two sets of traffic lights. The probability that the bus has to stop at a set of lights is 0.6.

   (a) What is the probability that the bus does not have to stop at a set of traffic lights?

(b) Copy the tree diagram below and add the correct probabilities to each branch.



(c) What is the probability that the bus gets to school without having to stop at a traffic light?

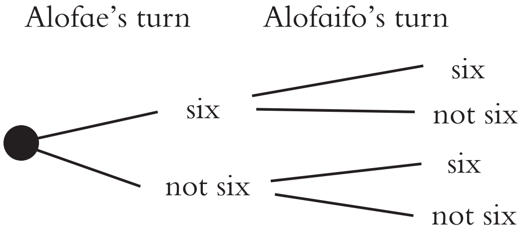
(d) What is the probability that the bus stops at both sets of traffic lights?

(e) What is the probability that the bus stops at one set of traffic lights?

2. Two girls are playing a game. They take it in turns to start throwing a dice. Before they start the game they must throw a six. Alofae starts first.

(a) What is the probability of throwing a six?

(b) Copy the tree diagram and add the appropriate probabilities to each branch. Also calculate the probability of each outcome show on the tree diagram.



(c) Find the probability that:

(i) both girls start the game on their first throws.

(ii) only one of them starts the game on their first throw.

(iii) neither of them starts the game on their first throw.

3. Draw a tree diagram to show the possible outcomes when two coins are tossed. Include the probabilities on your tree diagram.

Find the probability of obtaining:

(a) two heads.

(b) no heads.

(c) only one head.

4. Paulo travels to Apia on the early bus. The probability that he arrives late is $\frac{1}{10}$.

   He catches the bus on two consecutive days.

   What is the probability that he arrives:

   (a) on time on both days?

   (b) on time on at least one day?

   (c) late on both days?

5. When Sina's phone rings the probability that the call is for her is $\frac{3}{4}$.

   (a) What is the probability that a call is not for Sina?

   (b) Draw a tree diagram that includes probabilities to show the possible outcomes when the phone rings twice.

   (c) Find the probabilities that:

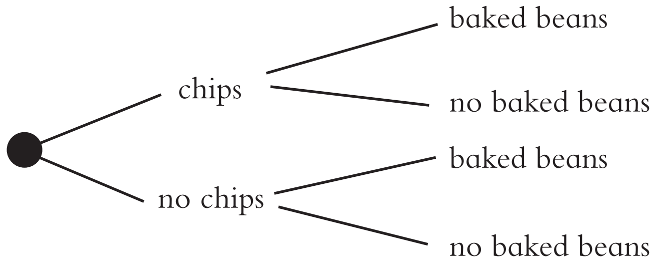
      (i) both calls are for Sina.

      (ii) only one call is for Sina.

      (iii) neither call is for Sina.

6. In a restaurant the probability that a customer has chips with their meal is 0.9 and the probability that they have baked beans is 0.6.

   (a) Copy and complete the tree diagram below.



   (b) What is the probability that a customer has:

      (i) both chips and beans?

      (ii) chips but not beans?

      (iii) neither chips nor beans?

7. To be able to drive a car you must pass both a theory test and a practical driving test. The probability of passing the theory test is 0.8 and the probability of passing the practical test is 0.6.

   (a) What is the probability of failing:

      (i) the theory test?

      (ii) the practical test?

(b) What is the probability that someone:

  (i) passes both tests?

  (ii) fails both tests?

8. Matiu and Mata play squash together. The probability that Mata wins is 0.52.

   (a) Find the probabilities that, out of two games:

      (i) Mata wins two.

      (ii) Matiu wins two.

      (iii) they win one each.

   (b) Which of the outcomes is the most likely?

9. Veronica calls for her friends, Kate and Fiona. The probability that Kate is not ready to leave is 0.2 and the probability that Fiona is not ready is 0.3.

   Use suitable tree diagrams to find the probability that:

   (a) both Fiona and Kate are ready to leave.

   (b) one of them is not ready to leave.

   (c) Kate is not ready to leave on two consecutive days.

   (d) Kate is ready to leave on two consecutive days.

10. A die has 6 faces of which 3 are green, 2 yellow and 1 red.

    Find the probabilities of the following outcomes if the die is rolled twice:

    (a) both faces have the same colour.

    (b) both faces are red.

    (c) neither face is green.

11. (a) Draw a tree diagram to show the possible outcomes when a coin is tossed three times.

    (b) Find the probability of obtaining:

      (i) 3 heads or 3 tails.

      (ii) at least 2 heads.

      (iii) exactly one tail.

12. On average Alisia comes to dinner on 2 days out of every 5. If Alisia comes to dinner, the probability that that we have fish is 0.7.

    If Alisia does not come to dinner, the probability that we have fish is 0.4.

    (a) Draw a tree diagram to illustrate this information.

    Write the appropriate probability on each branch.

    (b) What is the probability that we will have fish for dinner tomorrow?

Unit 5: ANSWERS — TRIGONOMETRY

Section 5.1 Solving Problems With 2-Dimensional Figures

(Pg. 7) Working with Pythagoras' Theorem

1. (a) 7.14 (b) 7.07 (c) 5.39
   (d) 7.75 (e) 10.95 (f) 13.89
2. (a) 14.14 (b) 1.94 (c) 1.29 (d) 3.12
3. (a) Yes (b) No (c) Yes (d) No
4. (a) 3.46 m (b) 1.73 m (c) 5.69 m
5. 142.13 cm
6. 332.75 cm

(Pg. 10) Finding Lengths in Right-Angled Triangles

1. (a) 5.14 cm (b) 11.82 cm (c) 5.13 cm
   (d) 6.06 cm (e) 9 cm (f) 8.21 cm
2. (a) 3.71 m (b) 1.50 m
3. (a) 1.73 m (b) 1.21 m (c) 1 m
4. 0.60 m
5. 143.4 m
6. (a) 386.4 km (b) 103.5 km
7. (a) 103. 9 km (b) 60 km
8. 20.5 m to 35.3 m
9. (a) 1.88 m (b) 2.92 m
10. 124.5 cm

(Pg. 13) Finding Angles in Right-Angled Triangles

1. (a) 53.1° (b) 71.6° (c) 75.5°
   (d) 47.0° (e) 33.1° (f) 18.6°

2. 60°

3. 11.5°

4. (a) 21.8° (b) 68.2°

5. (a) 48.2° (b) 11.18°

6. 040°

7. 054°

8. (a) $\alpha = 33.7°$, $\beta = 19.4°$ (b) 7.21 m, 10 m

9. 5.74°

10. (a) 12.37 cm (b) 72.08°

(Pg. 16) Solving Problems with Trigonometry

1. 8.82 m

2. 12.50 m

3. 7.13°

4. 1.03°

5. (a) 381.6 m (b) 1.91°

6. 7.85 m

7. 7.20 m

8. (a) 8.96 m (b) 38.5° (c) 6.72 m

(Pg. 17) Dealing with Angles Larger than 90°

1. (a) 25.98 cm (b) 37.59 cm (c) 20.78 cm (d) 49.24cm

2. (a) 9.33 m (b) 7.5 m

Section 5.2 Graphing Trigonometric Functions

(Pg. 19) Expressing Trigonometric Ratios as Fractions

1.

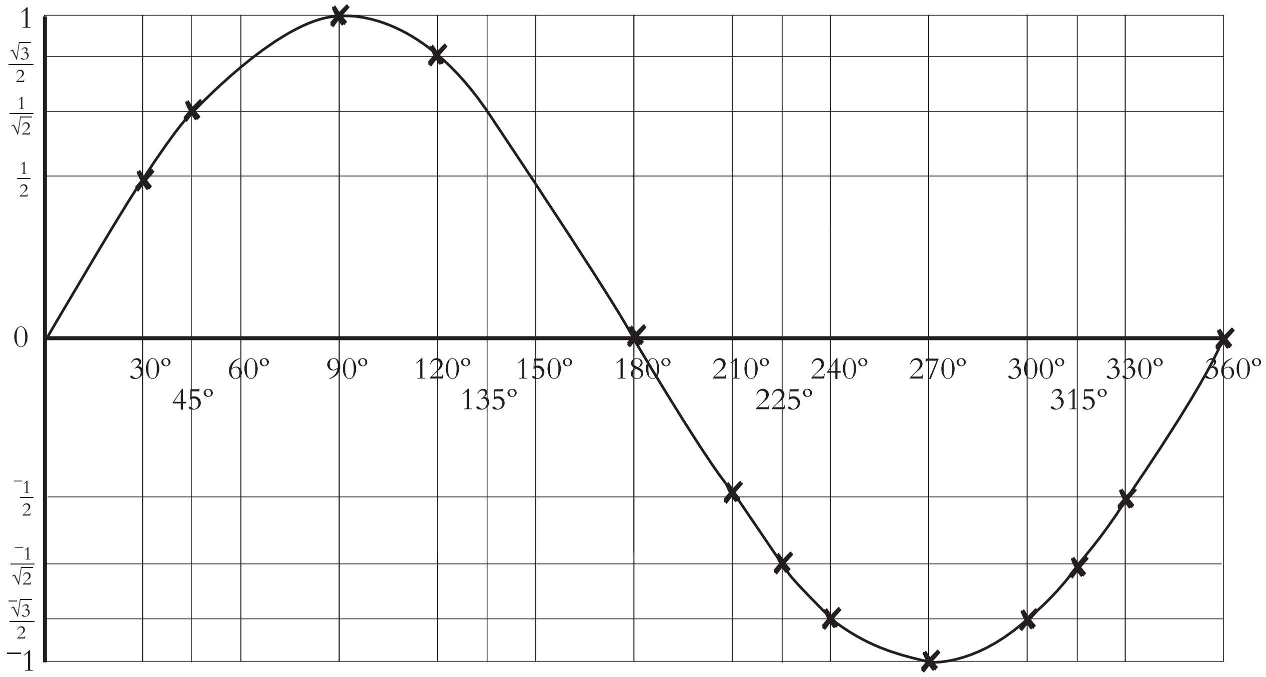
	Sin θ	Cos θ	Tan θ
30º	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45º	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60º	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1}$

2.

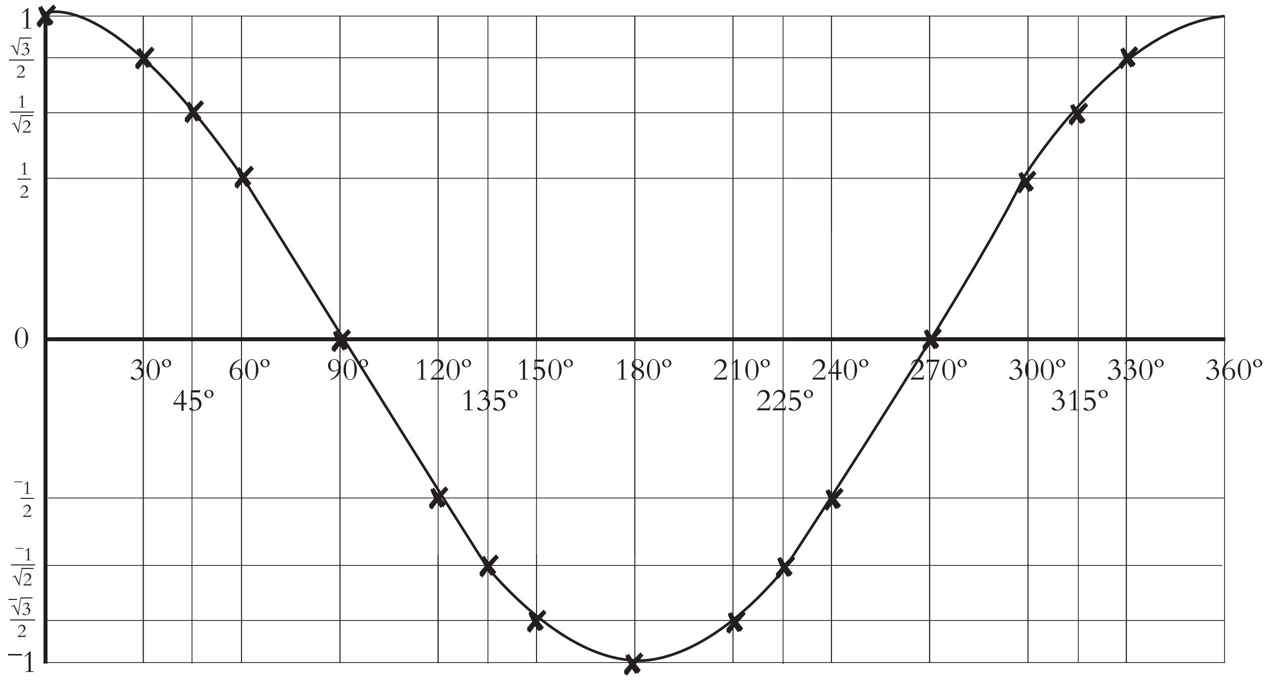
	Sin θ	Cos θ	Tan θ
0º	0	1	0
30º	0.5	0.8660	0.5773
45º	0.7071	0.7071	1
60º	0.8660	0.5	1.7320
90º	1	0	—

3. (a) 0.5 (b) −0.5 (c) 0.7071 (d) −0.7071
(e) 0.8660 (f) −0.8660 (g) 0.5773 (h) −0.5773
(i) 1.7320 (j) −1.7320

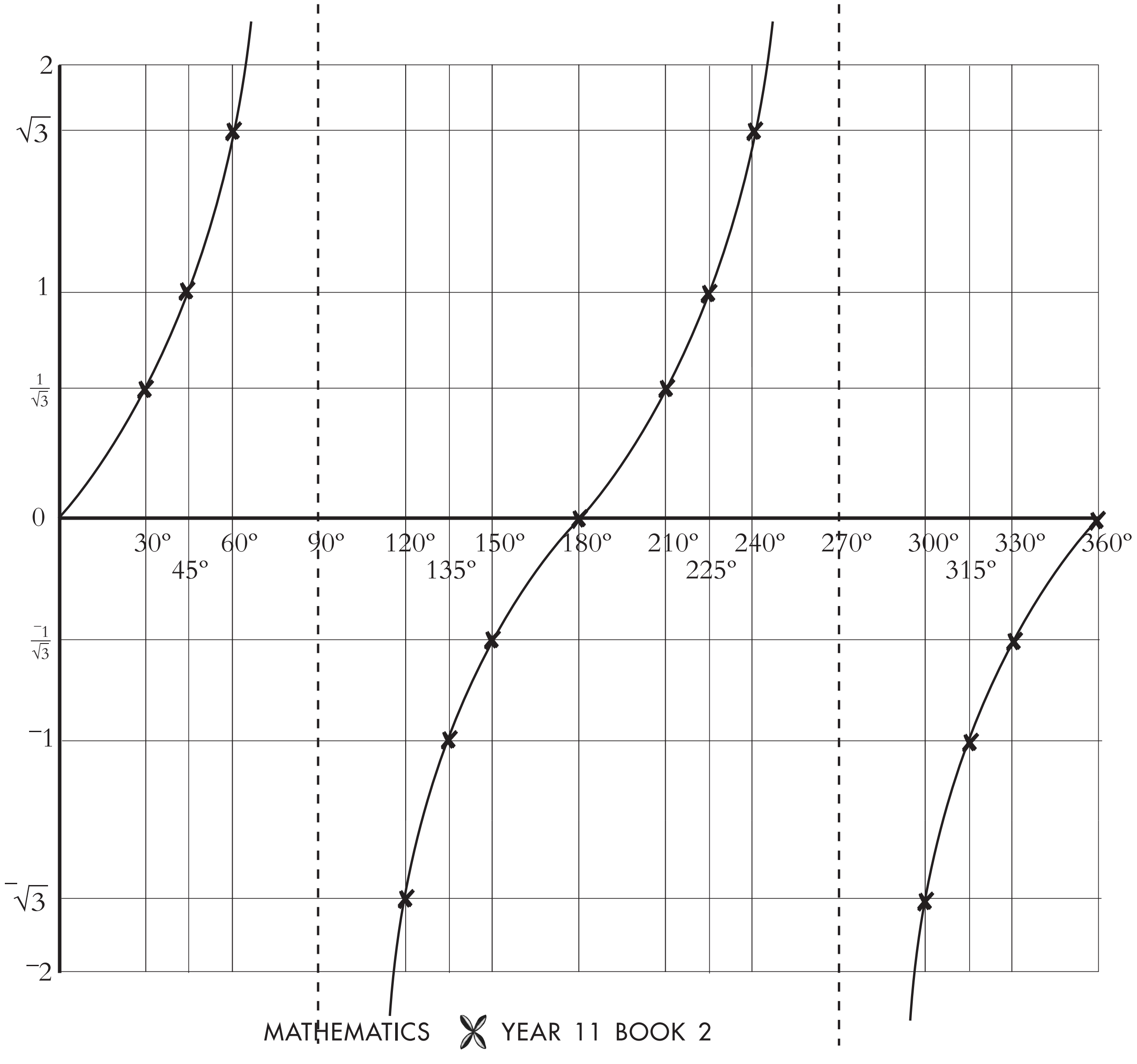
(Pg. 20) Drawing the Sine Curve



(Pg. 20) Drawing the Cosine Curve



(Pg. 21) Drawing the Tangent Curve



Unit 6: ANSWERS — GEOMETRY

Section 6.1 Using Bearings To Describe Direction And Plot Courses

(Pg. 24) Using Bearings to Describe Direction and Plot Courses

1. (a) Fereita (b) Semisi (c) Eseta (d) Robin
(e) Mose (f) Robin (g) NE

2. (a) 100° (b) 155° (c) 177° (d) 355°
(e) 207° (f) 180°

3. (a) (i) 135° (ii) 090° (iii) 045° (b) 230 m

4. (a) 128° (b) 267° (c) 257° (d) 317°
(e) 100°

5. (a) 535 m, 450 m (b) 38 m (c) 348°, 67 km
(d) 293° (e) 073°, 465 m

Section 6.2 Constructing Scale Drawings

(Pg. 27) Drawing Plans

1. (a) 6 m by 6 m (b) 32.5 m^2 (c) 10.5 m

2. (a) 3.6 m, 2.4 m (b) 60 cm by 60 cm (c) 60 cm by 180 cm
(d) 3.78 m^2

4. (a) 3 m by 3.25 m (b) 1.75 m (c) 1.125 m^2
(d) 9.75 m^2

5. (b) 4.2 m, 5.8 m

6. (a) 4.8 m (b) 3.6 m (c) 2.5 cm by 1.875 cm
(d) 0.75 cm

7. (a) 8 cm by 10 cm (b) 16 cm by 20 cm (c) 4 cm by 5 cm

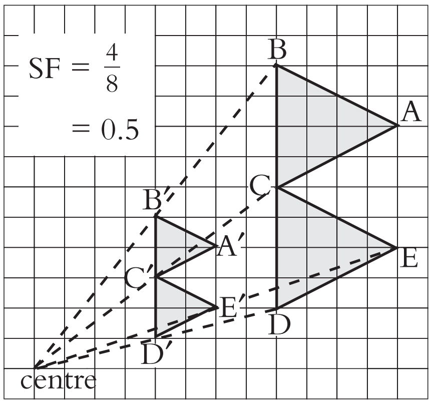
8. (a) 1 : 131.25 m^2, 2 : 300 m^2, 3 : 162.5 m^2, 4: 275 m^2, 5 : 475 m^2

9. (a) 5 m (b) 16 cm

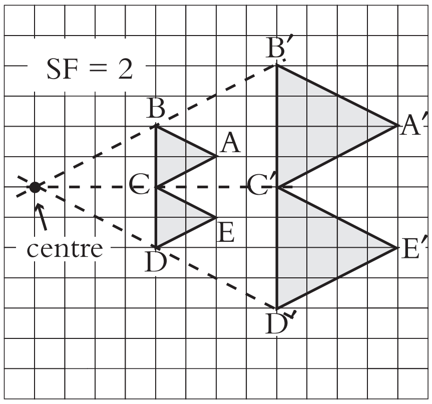
10. (a) 2 cm (b) 13.4 cm

(Pg. 33) Working with Enlargements

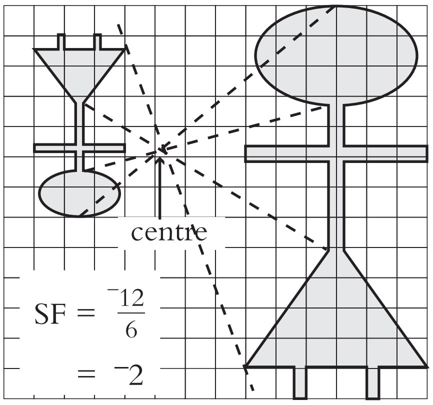
1. (a)



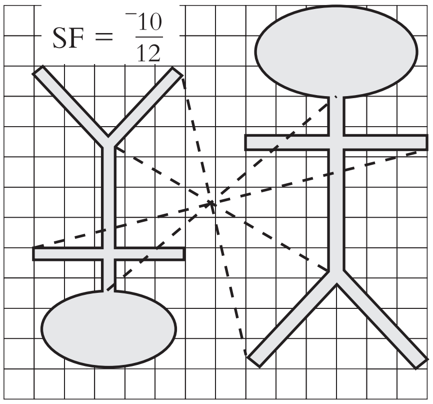
(b)



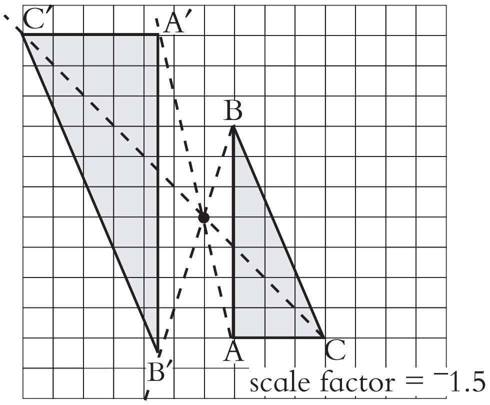
2. (a)



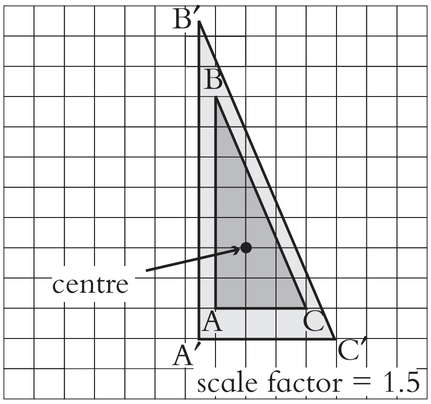
(b)



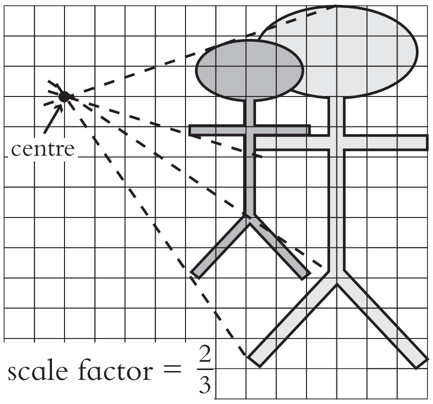
3. (a)



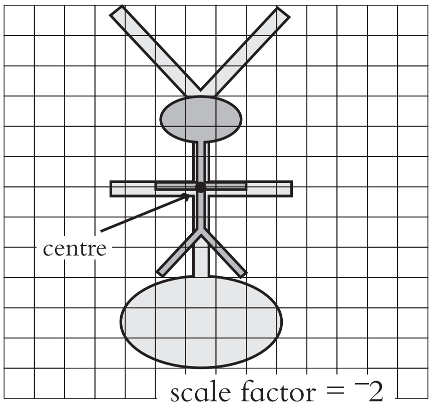
(b)



4. (a)



(b)



Section 6.3 Using The Properties Of Circles To Solve Problems

(Pg. 36) Angles in Semi-circles

1. (a) $a = 90^\circ$, $b = 65^\circ$ (b) $a = 72^\circ$, $b = 90^\circ$
   (c) $a = 90^\circ$, $b = 76^\circ$, $c = 90^\circ$, $d = 74^\circ$
   (d) $a = 90^\circ$, $b = 58^\circ$, $c = 90^\circ$, $d = 32^\circ$
   (e) $a = 90^\circ$, $b = 49^\circ$, $c = 60^\circ$, $d = 60^\circ$, $e = 30^\circ$, $f = 30^\circ$, $g = 120^\circ$
   (f) $a = 40^\circ$, $b = 100^\circ$, $c = 50^\circ$, $d = 50^\circ$, $e = 80^\circ$
   (g) $a = 44^\circ$, $b = 44^\circ$, $c = 46^\circ$, $d = 46^\circ$, $e = 88^\circ$, $f = 44^\circ$
   (h) $a = 70^\circ$, $b = 40^\circ$, $c = 140^\circ$, $d = 20^\circ$, $e = 20^\circ$

2. (a) $a = 90^\circ$, $b = 65^\circ$ (b) $a = 74^\circ$
   (c) $a = 90^\circ$, $b = 90^\circ$, $c = 50^\circ$, $d = 75^\circ$
   (d) $a = 10^\circ$, $b = 170^\circ$

3. (a) $a = 48^\circ$, $b = 84^\circ$, $c = 42^\circ$, $d = 42^\circ$, $e = 96^\circ$
   (b) $a = 75^\circ$, $b = 75^\circ$, $c = 15^\circ$, $d = 15^\circ$, $e = 150^\circ$
   (c) $a = 69^\circ$, $b = 69^\circ$, $c = 42^\circ$, $d = 69^\circ$, $e = 69^\circ$
   (d) $a = 28^\circ$, $b = 124^\circ$, $c = 70^\circ$, $d = 40^\circ$, $e = 16^\circ$

4. (a) $a = 30^\circ$, $b = 70^\circ$, $c = 70^\circ$, $d = 80^\circ$
   (b) $a = 110^\circ$, $b = 140^\circ$

(Pg. 40) Using the Five Proofs

1. (a) 30° (b) 120°
   (c) $c = d = 35°$ (d) 146°
   (e) $f = 90°$, $g = 55°$ (f) $x = y = 43°$
   (g) $a = 65°$, $b = 25°$, $c = 25°$, $d = 65°$
   (h) $a = 27°$, $b = 126°$, $c = 63°$

2. (a) $a = 120°$, $b = 75°$ (b) $c = 149°$, $d = 123°$
   (c) $a = 55°$, $b = 125°$ (d) $c = 140°$
   (e) $a = 48°$, $b = 75°$ (f) $a = 75°$, $b = 100°$
   (g) $a = 85°$, $b = 30°$ (h) $x = 160°$

3. (a) 50° (b) 12.5° (c) 40°

Unit 7: ANSWERS — STATISTICS

Section 7.1 Distinguishing Different Types Of Data

(Pg. 43) Discrete and Continuous Data

(a) continuous

(b) discrete

(c) continuous

(d) continuous

(e) discrete

(f) continuous

(g) discrete

(Pg. 44) Grouped and Ungrouped Data

1.

Score	Tally	Frequency
$20 < s \leq 30$	||	2
$30 < s \leq 40$	||	2
$40 < s \leq 50$	𝍸	5
$50 < s \leq 60$	𝍸 |||	8
$60 < s \leq 70$	𝍸 |	6
$70 < s \leq 80$	||||	4
$80 < s \leq 90$	||	2
$90 < s \leq 100$	|	1

2. Note how the data is grouped.

Height (cm)	Tally	Frequency
$155 \leq h < 160$	||	2
$160 \leq h < 165$	𝍸	5
$165 \leq h < 170$	𝍸 ||	7
$170 \leq h < 175$	𝍸	5
$175 \leq h < 180$	|	1

Section 7.2 Working With Data

(Pg. 46) Mean, Median, Mode and Range

1. (a) mean = 5 median = 4.5 mode = 3 range = 8
   (b) mean = 9.9 median = 9.5 mode = 7 range = 11
   (c) mean = 16.25 median = 12.5 mode = 17 range = 10
   (d) mean = 107.6 median = 108 mode = 108 range = 13
   (e) mean = 63 median = 64 mode = 61 range = 10
   (f) mean = 21.75 median = 21.5 mode = 16 range = 14

2. (a) 6.8 (b) 6.75 (c) 6 (d) $5\frac{1}{2}$

3. (a) (i) 4.875 (ii) 3.5 (iii) $3
   (b) (i) mean (ii) mode
   (c) range = $13

4. (a) Fred: mean = 20.8, range = 2. Harry: mean = 24.2, range = 24.
   (b) Fred (c) Harry

5. (a) A: mean = 15.9, median = 20, mode = 20
   B: mean = 16.9, median = 17, mode = 17
   (b) Mode suggests A
   (c) Mean suggests B
   (d) Range: A = 15, B = 3.

6. (a) mean = 68, median = 68.5
   (b) His mean increases to 68.4.
   (c) The median. It increases from 68.5 to 70, whereas the mean only increases by 0.4.

7. (a) Because the mode and median are very low.
   mode = 0 median = 1.
   (b) It looks as if Felise is not a very good fisherman.
   mean = 2.6 range = 18
   (c) 15 fish

8. (a) 268.4 cars
   (b) The mean decreases.

9. (a) mean = 2.035, median = 2, mode = 2.
   (b) Either median or mode (whole numbers).

10. The mean will increase.

11. (a) 19°C (b) Archangel (c) 27°C

12. 225 grams

(Pg. 51) Finding the Mean from Tables and Tally Charts

1. mean = 1.25

2. mean = 1.93

3. mean = 4.08

4. mean = 3.56

5. (a) mean = 1.95 (b) 22 times

6. mean number = 1.15 buses

7. mean = 2.30

8. (a) 6 (b) 2.04

9. Missing frequencies are 1, 5, 0.
Missing number of tickets are 0, 20, 21, 10, 0.
Mean = 2.8 (1 d.p.)

10. (a)

Weight Range (w)	Tally	Frequency
$30 < w \le 40$	IIII	4
$40 < w \le 50$	𝍸 II	7
$50 < w \le 60$	𝍸 II	7
$60 < w \le 70$	𝍸 III	8
$70 < w \le 80$	I	1
$80 < w \le 90$	III	3

(b) class $60 \le w < 70$

11. (a) 3

(b) frequency = 21, total = 48, mean = 2.29.

(c) The number of children per family has decreased on average (2.29<2.7), and there is less variation from family to family (today's range is 3, but in 1960 it was 7).

(Pg. 55) Calculations with the Mean

1. mean = 161 55

2. Mean = 2

3. Mean = 4

4. Mean = 60.15 kg

5. 7
6. 84%
7. $6000
8. 9.5
9. Mean ≈ 4.47
10. 320

(Pg. 58) Mean, Median and Mode for Grouped Data

1. (a) 32.09 (b) 33.83 (c) 30 – 39

2. (a) 40.3 (b) 40.7 (c) $40 \leq w < 45$

3. (a) Yes (b) 20.54 (c) 20.95
   (d) Median is greater than the mean.

4. (a) No (b) median = 0.72, mean = 0.78
   (c) The mean is the largest.

5. 9.6 years

6. (a) 11.95 (b) 12.6

7. (a) 9.65 (b) 10.42 (c) 11–15

8. (a) (i) 26.78 (ii) 27.66 (iii) 21–30
   (b) (i) 21.5 (ii) 22.75 (iii) 21–30
   (c) The second class have a lower mean but similar range.

9. (a) 0–$1.00 (b) $1.44

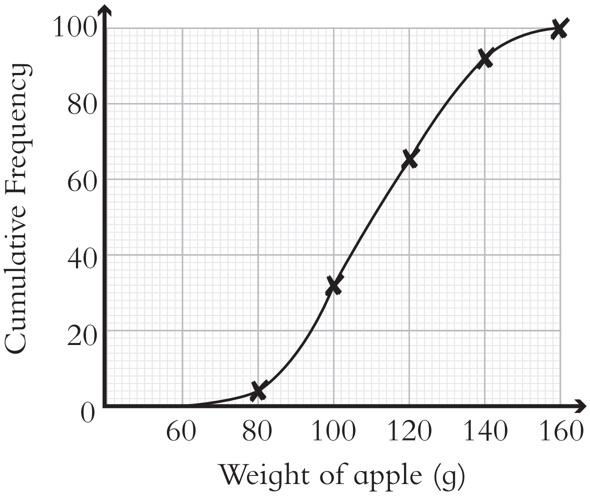
10. (a) 21, 7, 2
    (b) People would spend more time watching television than in summer.
    (c) 23.83 (24 hours)

(Pg. 65) Cumulative Frequency

1. (a)

Weight in grams (w)	Cumulative Frequency
$60 < w \leq 80$	4
$80 < w \leq 100$	32
$100 < w \leq 120$	65
$120 < w \leq 140$	2
$140 < w \leq 160$	100

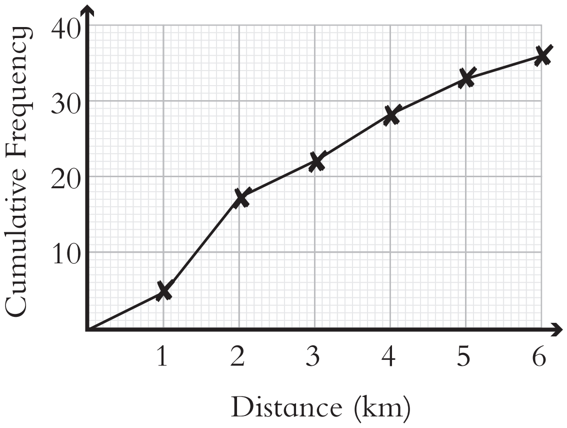
Median = 111, Inter-quartile range = 33.



(b)

Distance in km(d)	Cumulative Frequency
$0 < d \leq 1$	5
$1 < d \leq 2$	17
$2 < d \leq 3$	22
$3 < d \leq 4$	28
$4 < d \leq 5$	33
$5 < d \leq 6$	36

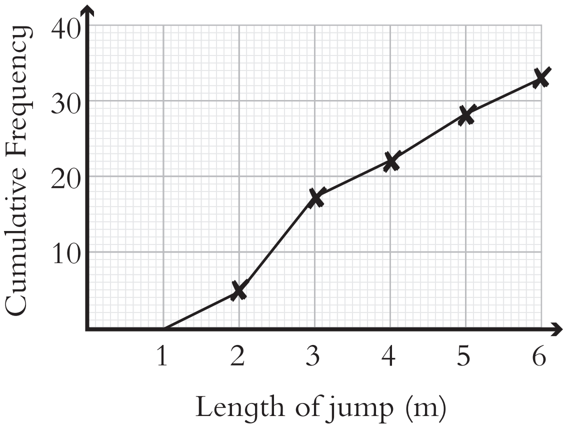
Median = 2.2, Inter-quartile range – any answer between 2 and 3 is acceptable.



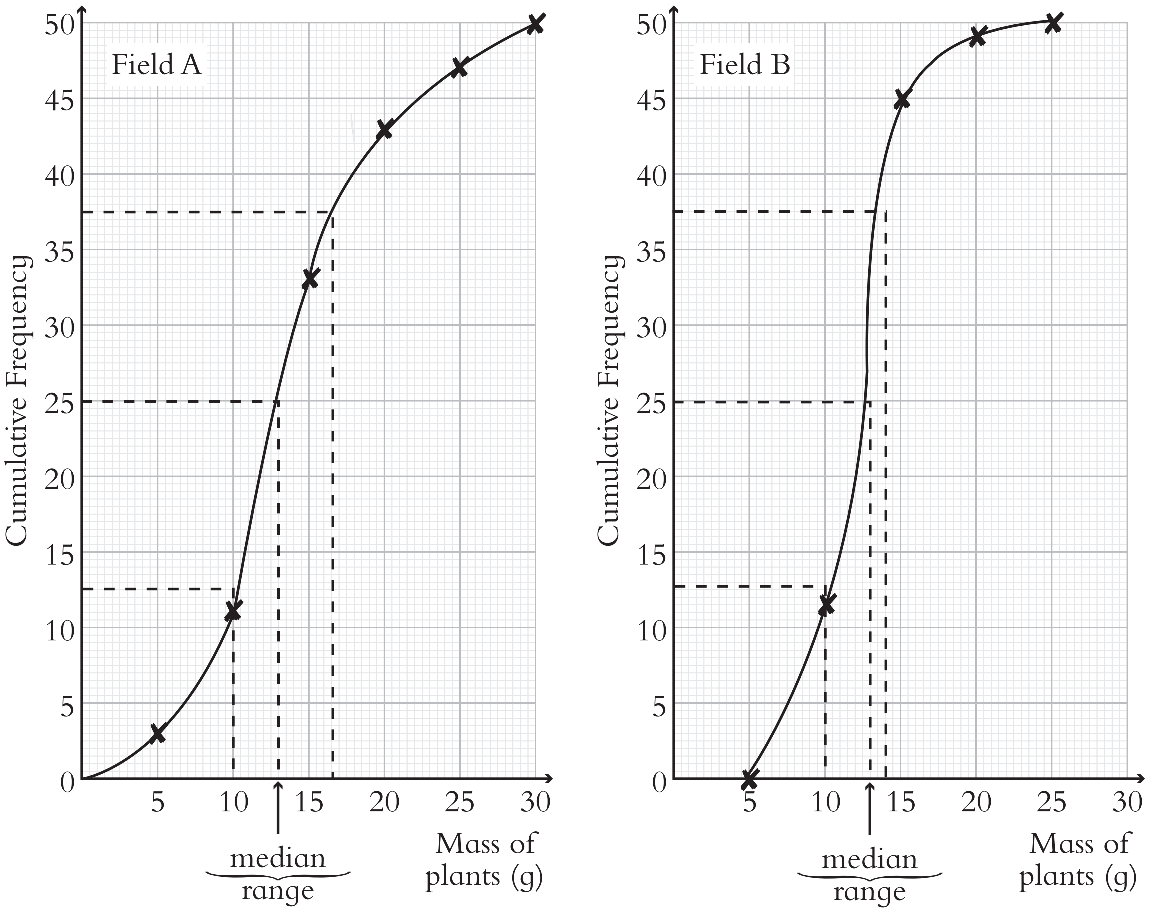
(c)

Distance in metres (d)	Cumulative Frequency
$1 < d \leq 2$	5
$2 < d \leq 3$	17
$3 < d \leq 4$	22
$4 < d \leq 5$	28
$5 < d \leq 6$	33

Median = 3, Inter-quartile range – any answer between 2 and 2.5 is acceptable.



2. (a)



(b) The median of field A is 13 g, the median in field B is 12 g. The inter-quartile range for field A is around 7, for B it is around 4.

(c) Field B is more reliable than field A (its inter-quartile range is narrower), although it is less productive in 50% of the cases (its median is lower than the median of field B).

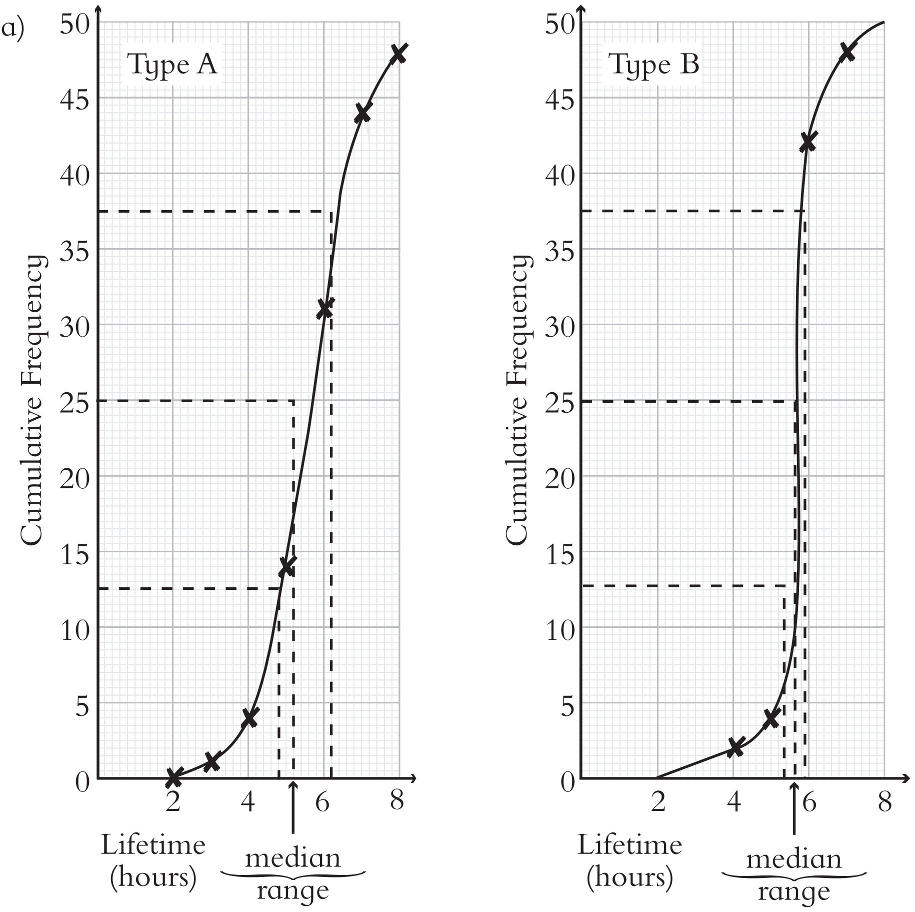
3. (a) See diagram on opposite page.

Median for type A = 5.6, Inter-quartile range for type A = 1.2.

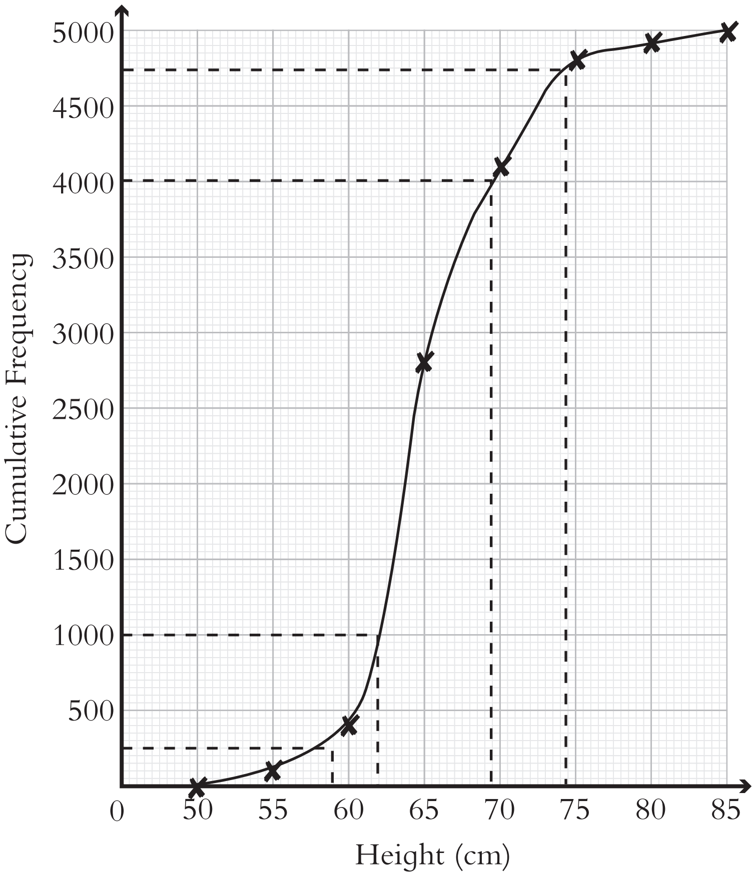
Median for type B = 5.8, Inter-quartile range for type B = 0.8.

(b) Type B – although both types have quite similar medians, type B is more predictable (its inter-quartile range is narrower) and most of the time its lifetime is above 5.

3. (a)

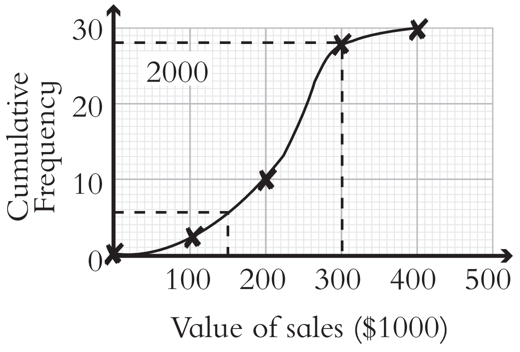


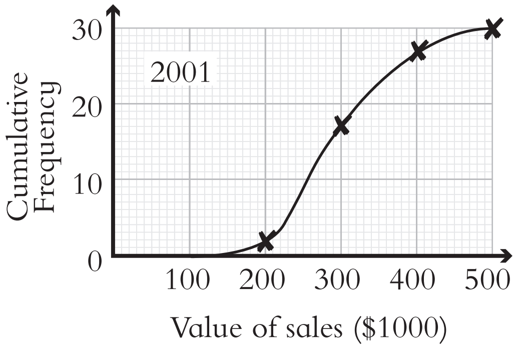
4. The heights of children in each category (to the nearest cm) are:



Very tall - $75 < h \le 85$
Tall - $70 < h \le 75$
Normal - $62 < h \le 70$
Short - $58 < h \le 62$
Very short - $50 < h \le 58$

5.





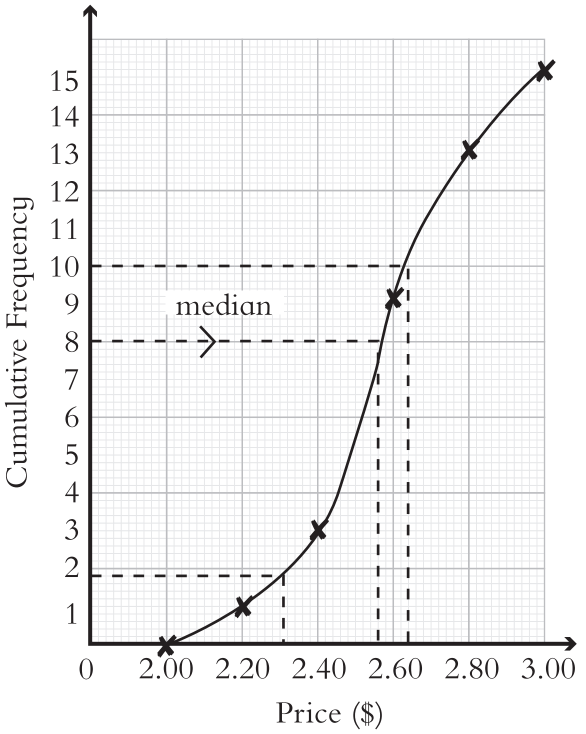
For 2000:

Bonus	Value of sales
$50	$300 < V \leq 400$
$250	$140 < V \leq 300$
$500	$0 < V \leq 140$

For 2001:

Bonus	Value of sales
$50	$100 < V \leq 230$
$250	$230 < V \leq 400$
$500	$400 < V \leq 500$

6. (a)

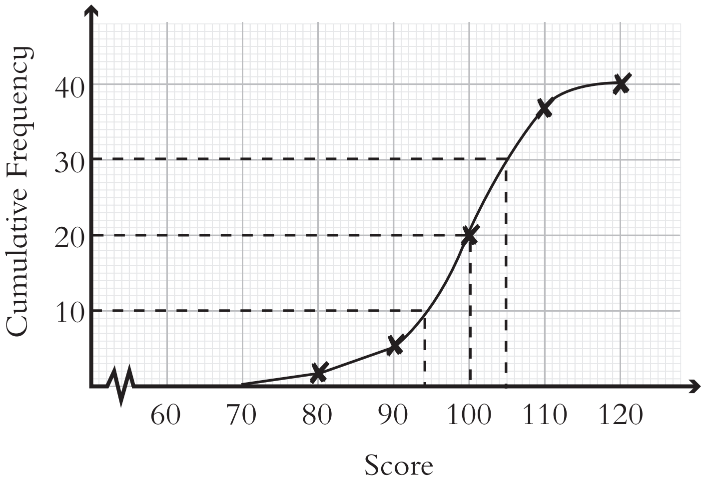


(i) 5 or 6 shops (ii) About $2.50 (iii) 2 shops

(iv) 8 shops (v) 5 or 6 shops

(b) The only exact answer is (iv). The other answers are estimates since they relate to prices for which we do not have exact information.

7. (a)



(b) (i) Laura's median = 100

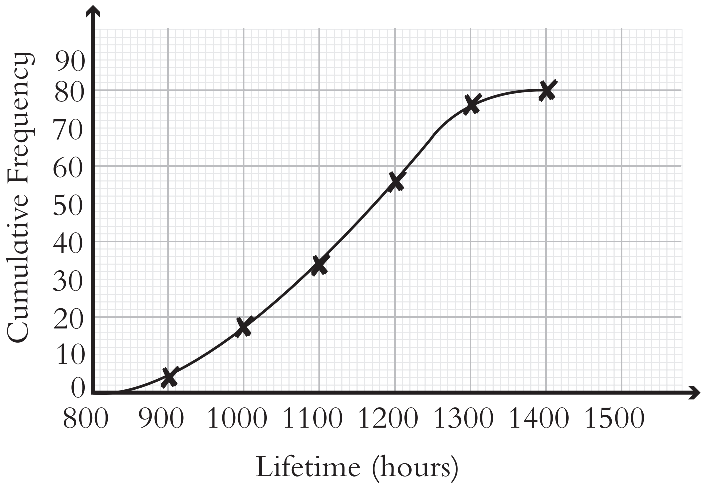
(ii) Inter-quartile range – between 94 and 105 = 11.

(c) (i) Simiona was the more consistent player because her inter-quartile range is lower.

(ii) Laura won most of the games. She scored less than 100 in 20 matches and more than 103 in only 16 matches, whereas Simiona scored more than 103 in 20 matches and her inter-quartile scores were quite consistent.

8. (a) Cumulative frequencies – 34, 56, 76, 80, 80.

(b)

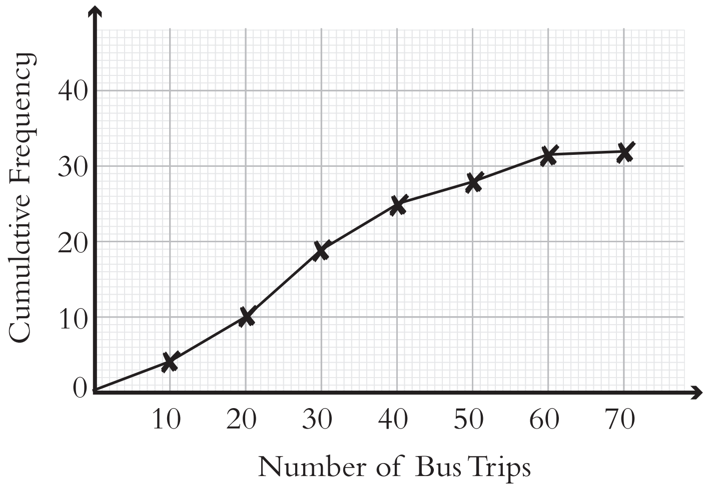


(c) 58 bulbs

(d) Inter-quartile range – between 1020hrs and 1210 hrs = 190 hrs

(e) The bulbs from the second sample are more reliable than those from the first.

9. (a) Cumulative frequencies 4, 11, 19, 25, 28, 32, 32.



(b) Median – around 30.

(c) 6 people.

(d) Both groups made the same number of trips.

Unit 8: ANSWERS — PROBABILITY

Section 8.1 Calculating Probabilities

(Pg. 73) Simple Probability

1. (a) 0.1 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$
2. (a) 0.2 (b) 0.4
3. 0.98
4. $\frac{4}{5}$
5. (a) $\frac{4}{7}$ (b) not to rain
6. 0.99
7. (a) $\frac{2}{5}$ (b) 12
8. (a) 0.6 (b) 0.9 (c) 0.1
9. (a) $\frac{9}{20}$ (b) $\frac{10}{11}$ (c) $\frac{9}{13}$
10. No. They may also have black eyes or green eyes.
11. 0.15
12. (a) C (b) B
13. (a) near to 0 (b) near to 1

(Pg. 76) Determining the Outcomes of Two Events

1. VC,VS, MC, MS, RC, RS
2. GG, RG, GR, RR
3. (a) Roy wins sit ups and Ben wins press ups.
   (b) BI, BR, BB, IB, IR, II, RB, RI, RR

(c) BI, BB, IB, II

(d) BB, BI, BR, BT, IB, II, IR, IT, RB, RI, RR, RT, TB, TI, TR, TT

4. BS, BT, BD, BB, ST, SD, SS, TD, TT, DD

5. CD, CB, CC, DB, DD, BB

6.

	F	M
H	HF	HM
T	TF	TM
C	CF	CM

(Pg. 78) Finding Probabilities Using Relative Frequency

4. $\frac{1}{4}$

5. $\frac{4}{5}$

6. (a) $\frac{2}{7}, \frac{7}{10}, \frac{1}{4}, \frac{1}{5}, \frac{1}{3}, \frac{2}{3}$ (b) Anatalia
(c) Rachel (d) Siaki

7. (a) $\frac{3}{8}$ (b) $\frac{5}{12}$ (c) $\frac{5}{24}$

8. (a) $\frac{4}{5}$ (b) 96

(Pg. 81) Finding the Theoretical Probability of One Event

1. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{13}$ (e) $\frac{4}{13}$

2. (a) $\frac{13}{54}$ (b) $\frac{13}{54}$ (c) $\frac{2}{27}$ (d) $\frac{2}{27}$ (e) $\frac{8}{27}$

3. (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}$

4. (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$ (e) $\frac{1}{4}$

5. $\frac{3}{8}$

6. (a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$ (e) $\frac{9}{49}$
(f) $\frac{39}{49}$ (g) $\frac{10}{49}$

7. (a) $\frac{9}{25}$ (b) $\frac{6}{25}$ (c) $\frac{3}{5}$ (d) $\frac{16}{25}$

8. (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{5}$

9. (a) $\frac{3}{10}$ (b) $\frac{2}{9}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

(Pg. 84) Finding the Theoretical Probability of Two Events

1. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$

2. (a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

   (e) $\frac{1}{2}$

3. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{9}$ (iii) $\frac{1}{2}$ (iv) $\frac{5}{18}$

   (v) $\frac{5}{18}$

   (b) 7

4. (b) (i) $\frac{1}{9}$ (ii) $\frac{1}{3}$ (iii) $\frac{8}{9}$

   (c) $\frac{1}{9}$

5. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$

6. (a) $\frac{1}{8}$ (b) $\frac{5}{8}$ (c) 5

7. (a) 6 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$

8. (a) 400 (b) $\frac{1}{400}$ (c) $\frac{1}{20}$ (d) $\frac{1}{25}$

9. (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$

10. (a) (i) 1, 2; 1, 3; 2, 2; 2, 3; 3, 2; 3, 3 (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$

    (b) (i) 9

    (ii) If the new card is drawn, the total will always be greater than 5.

    (iii) Because there are now more possible outcomes.

(Pg. 89) Using Tree Diagrams

1. (a) 0.4

   (b) $0.6 \times 0.6 = 0.36$; $0.6 \times 0.4 = 0.24$; $0.4 \times 0.6 = 0.24$; $0.4 \times 0.4 = 0.16$

   (c) 0.16 (d) 0.36 (e) 0.48

2. (a) $\frac{1}{6}$ (c) (i) $\frac{1}{36}$ (ii) $\frac{5}{18}$ (iii) $\frac{25}{36}$

3. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$

4. (a) $\frac{81}{100}$ (b) $\frac{99}{100}$ (c) $\frac{1}{100}$

5. (a) $\frac{1}{4}$

(c) (i) $\frac{9}{16}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{16}$

6. (b) (i) 0.54 (ii) 0.36 (iii) 0.04

7. (a) (i) 0.2 (ii) 0.4
(b) (i) 0.48 (ii) 0.08

8. (a) (i) 0.2704 (ii) 0.2304 (iii) 0.4992
(b) (iii)

9. (a) 0.56 (b) 0.38 (c) 0.04 (d) 0.64

10. (a) $\frac{7}{18}$ (b) $\frac{1}{36}$ (c) $\frac{3}{4}$

11. (b) (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{8}$

12. (b) 0.52